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A MULTI-MARKET APPROACH TO MEASURING THE CYCLE

by

Kenneth W. Clements[1] and Grace Gao[2]

Abstract

At any given moment there are numerous indicators of the state of an economy or sector. Frequently, these signals are divergent, for example, some may point to an expansion, while others to a contraction. We consider how best to combine such conflicting information into an overall index of economic conditions. This index plays the role of the "underlying cycle" and has the property of minimising the distortionary impact of noise of the n individual signals. This is essentially the panel regression approach of Stock and Watson (2010). We elaborate and evaluate this rich approach with reference to stochastic index number theory, and suggest new interpretations, modifications and extensions, illustrated using world prices of six important metals.

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1. Introduction

Consider a sector of the economy made up of a number of distinct markets. An example is the metals sector made up of aluminium, copper, lead, nickel, tin and zinc. Each market is subject to a large number of shocks, some common to all markets, others market specific. Common shocks to the metals sector could be, for instance, a surge in construction in China substantially adding to world demand for metals; major central banks embarking on coordinated quantitative easing leading to a large increase in global liquidity; or a rise in energy prices depressing the world economy. Market-specific shocks could include, for instance, technological breakthroughs that make lower-grade ore deposits commercially viable; strikes in major supplying countries; or natural disasters that disrupt production of certain metals. If there is a reoccurring pattern of common shocks, then it is likely to result in price cycles.

The question we consider is, how can we best use this disaggregated price information to identify the underlying cycle for a sector as a whole? The problem of measuring the cycle in a multi-market context can be thought of as a statistical one: Although the underlying cycle is unobservable, each market provides a noisy reading brought about by market-specific shocks. The statistical problem is to minimise the distortionary impact of the noise to estimate the underlying cycle by employing some form of averaging. This averaging can be conveniently formulated as a type of panel regression model using data across markets and time. This is a particularly rich framework as it provides point estimates of turning points, the corresponding standard errors and, with some additional assumptions, the whole probability distribution. The dispersion of this distribution is directly related to the extent to which the individual markets are idiosyncratic. When, for example, some prices experience a longer expansion than others, there is substantial diversity across markets. Here, the standard error of the estimated peak of the underlying boom would be large, reflecting the greater uncertainty associated with the nature of this phase of the cycle for the sector as a whole. This probabilistic approach means that hypothesis testing can be carried out to study questions such as: Is the duration of one boom longer than another? Are booms longer than slumps? And following a peak, do prices initially collapse and then tail off as they approach the trough? At a fundamental level, this approach is equivalent to the stochastic approach to index numbers, which emphasises index numbers as means of prices (or quantities) and applies the theory of the sampling distribution of the mean to the index. In other words, the multi-market approach to measuring the common cycle can be considered a branch of index number theory, something not previously recognised. We show how this is a useful approach for measuring cycles.

As GDP is the sum of the value added in each sector, the multi-market approach is applicable for measuring the cycle for the economy as a whole. In a pioneering study, Stock and Watson (2010) develop this idea to dating the business cycle. We elaborate their approach by examining in detail the workings of their panel regression model, and highlight as the source of the estimation error of the underlying cycle the extent to which the disaggregated variables display disparate cyclical behaviour as measured by the dispersion of their turning points. We deal with the estimation of the turning points of the cycle (the dates of the peaks and troughs), as do Stock and Watson (2010), but our approach is equally applicable to other characteristics of the cycle such as the amplitude and nature of the path between turning points. Turning points of the cycle are dated by the Bry-Boschan (1971) algorithm in this paper.[1]

In addition to Stock and Watson (2010), of relevance to our work is the paper by Harding and Pagan (2006) on the degree of synchronization of cycles across sectors and countries. They develop an algorithm to extract a common cycle based on non-parametric techniques. Whether any period t is a turning point is determined by calculating the median distance between t and the turning points of n individual series. The date at which the median is closest to zero is identified as the aggregate turning point. This approach contrasts with that of Stock and Watson (2010), which, as mentioned above, is based on a regression model. It is also relevant to mention dynamic factor models as a related branch of the literature (see Bai and Ng, 2008, and Stock and Watson, 2011). Here, a large number of macroeconomic variables are related to a smaller number of contemporaneous and lagged latent factors that represent the state of the underlying economy. When there is only one factor in such a model, its interpretation is clearly the common factor that codes the state of the whole economy. For example, that factor could be the latent growth if the original variables are expressed as growth rates. When there are multiple factors, however, their interpretation becomes more challenging.[2]

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[1] Alternative dating procedures include the filters of Baxter and King (1999) and Hodrick and Prescott (1997). Prominent studies of cycle measurement include Baxter and King (1999), Burns and Mitchell (1946), Bry and Boschan (1971), Canova (1994), Chauvet and Piger (2008), Hamilton (1989), Harding and Pagan (2006), Hodrick and Prescott (1997), King and Rebelo (1999) and Yogo (2008).

[2] The forecasting of turning points in economic variables discussed by Helperin (2010), Kling (1987), Wecker (1979), Zellner et al. (1990) and Zellner et al. (1991), among others, is also related to the dating and measurement of business cycles, but explicit links are yet to be fully developed.

The next section of the paper discusses in detail the meaning of the underlying cycle and the Stock and Watson (2010) approach, followed by an econometric analysis in Section 3. Section 4 is an illustrative application to the metals sector comprising the prices of six metals. Simulations are employed to examine the reliability of the approach in Section 5. The relationship between index numbers and cycle measurement is elaborated further in Section 6, and concluding comments are given in Section 7.

2. The Underlying Cycle and the Stock and Watson Approach

The business cycle could be dated on the basis of the trajectory of a single variable such as GDP whereby an algorithm is used to identify local turning points as peaks or troughs. An alternative approach, used by the National Bureau of Economic Research (NBER), is to examine a number of disaggregated macroeconomic measures of real economic activity (for example, personal income, employment and industrial production) for individual peaks and troughs and then aggregate the dates to date the cycle. As GDP is the aggregation of the underlying macroeconomic variables, the two approaches could be described as (i) aggregate then date; and (ii) date then aggregate. Stock and Watson (2010) compare these two approaches. In this section, we describe, interpret and extend their work.

Suppose we have monthly data for $n=3$ disaggregated macroeconomic variables. In the simplest possible case, over the course of the cycle the three variables all peak in the same month, so that it is obvious that the common peak is the peak for the economy as a whole. This situation is illustrated as Scenario 1 in column 2 of Table 2.1, where the common peak occurs in month 5. There can be no uncertainty regarding this peak. Next, consider the slightly more complex case in which variable 1 peaks in month 3, variable 2 peaks in month 5 and variable 3 peaks in month 7. In this case, the mean peak also occurs in month 5 (Scenario 2, column 3, Table 2.1). The important difference is that now there is some dispersion around the mean, as measured by the standard deviation of 2 months. Moreover, there is now estimation uncertainty regarding the mean peak indicated by a standard error of 1.2 months. A rough 95% confidence interval for the peak in months is (3,7) which is substantial. Clearly, the diversity of behaviour in Scenario 2 represents a different economic situation from the first, where there is no diversity. Scenario 2 thus requires more caution, and perhaps even further investigation in declaring month 5 to be the peak. In Scenario 3 (column 4, Table 2.1) there is even greater dispersion and uncertainty where the peak occurs, even though the point estimate, month 5, is the same as that of Scenarios 1 and 2.

The model underlying the above analysis can be expressed as

(2.1) $y_i = \alpha + \varepsilon_i$, $i = 1, \ldots, n$ disaggregated variables,

where y_i is the peak (measured in terms of the month number) for variable i, α is the underlying peak, which is the same for all variables, and ε_i is a random disturbance term with zero expectation and constant variance. According to this model, the peak for each variable is made up of the common peak α plus a random term ε_i. Therefore, there are n noisy readings on the common peak. Thus, the estimation problem is to combine these n readings so as to minimise the noise. Under the stated properties of the disturbance term, the mean of n peaks of the disaggregated variables is the best linear unbiased estimator of the common peak.

Model (2.1) applies to one episode in which each of n variables has one peak. When we have a series of episodes, each with its own peak, model (2.1) can be written for episode e as $y_{ie} = \alpha_e + \varepsilon_{ie}$, $i = 1, \ldots, n$. It is possible that some of the underlying variables may lead or lag the common cycle in a consistent manner. If the peak for variable i occurs $\beta_i > 0$ months after the common peak, then to provide an unbiased reading on the common peak, we need to "shift" i's peak back β_i months by replacing y_{ie} on the left-hand side of the above equation with $y_{ie} - \beta_i$, so that it is now "in phase". If variable j leads, then $\beta_j < 0$, and we continue to subtract, so the net effect is to add the lead to synchronise it with the common trend. Thus, model (2.1) can be extended to deal with the "out-of-phase" variables by using $y_{ie} - \beta_i = \alpha_e + \varepsilon_{ie}$, or

(2.2) $y_{ie} = \alpha_e + \beta_i + \varepsilon_{ie}$, $i = 1, \ldots, n$ variables, $e = 1, \ldots, E$ episodes.

Each peak, y_{ie}, is now made up of the sum of three terms: a peak that is common to all n variable, α_e; a phase parameter specific to variable i, β_i; and a random disturbance term, ε_{ie}. As stated above, variables that lead (lag) have a negative (positive) value of β_i. As this model is subject to one additive degree of freedom, a natural identifying assumption is that the phase parameters sum to zero, $\Sigma_{i=1}^{n} \beta_i = 0$, so that the leading variables are balanced by the laggers, or the variables are synchronised on average. This same approach can equally be applied to dating the trough in each episode.

Stock and Watson (2010), using NBER business cycles as reference episodes and stratified sampling from n = 270 monthly variables, demonstrate their date-then-aggregate approach gives results reasonably close to the dates of peaks and troughs for the US economy as obtained by the NBER. The 95% confidence intervals for the turning points range from

±0.8 to ±1.8 months, indicating the estimates are relatively precise. Stock and Watson (2010) emphasise that the key benefit of their approach is that it provides standard errors for the turning points.

Stock and Watson (2010) treat model (2.2) as a fixed effects panel regression to be estimated by ordinary least squares. The problem they consider can also be thought of as an index number problem, in other words, how to best combine the turning points of the disaggregated variables into one number, namely, the peak or trough of the overall cycle. Traditionally, index number theory has been mostly deterministic involving the application of an aggregation formula (for example, Paasche, Laspeyres and Fisher) to yield one number, such as the overall rate of inflation. By contrast, the index number perspective of the Stock and Watson approach is a statistical one, leading to an estimate of the overall turning point that has a sampling distribution. The advantage of this approach is the ability to make probabilistic statements regarding the turning points and carry out hypothesis testing, which opens up new possibilities. On a formal level, the Stock and Watson approach is an application of stochastic index number theory. Some of the material that follows is motivated by that theory.[3]

3. The Econometrics of the Cycle

Model (2.2) can be written in vector form as

(3.1) $$\mathbf{y}_e = \alpha_e \iota + \boldsymbol{\beta} + \boldsymbol{\varepsilon}_e,\ e = 1,\ldots,E \text{ episodes},$$

where $\mathbf{y}_e = [y_{1e},\ldots,y_{ne}]'$ represents noisy readings from n individual series on the underlying turning point with date α_e; ι is a vector of n unit elements; and $\boldsymbol{\beta} = [\beta_1,\ldots,\beta_n]'$ is a vector of individual-specific effects representing lead/lag phase adjustments. Suppose the disturbance vector $\boldsymbol{\varepsilon}_e = [\varepsilon_{1e},\ldots,\varepsilon_{ne}]'$ has zero mean and a scalar covariance matrix $\boldsymbol{\Sigma}_e = \sigma^2\mathbf{I}$. The ordinary least squares estimators are

(3.2) $$\hat{\alpha}_e = \frac{1}{n}\sum_{i=1}^{n} y_{ie},\ e = 1,\ldots,E, \qquad \hat{\beta}_i = \frac{1}{E}\sum_{e=1}^{E}\left(y_{ie} - \hat{\alpha}_e\right),\ i = 1,\ldots,n,$$

which satisfy $\Sigma_{i=1}^{n}\hat{\beta}_i = 0$. The estimated common peak, $\hat{\alpha}_e$, is the mean of n peaks and the estimate of the i^{th} phase parameter, $\hat{\beta}_i$, is just the phase discrepancy for series i, $y_{ie} - \hat{\alpha}_e$, averaged over all episodes. The variances of these estimators are

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[3] There is currently a revival of interest in the stochastic approach. According to Diewert (2007), it is one of the four main approaches to index number theory—the fixed basket approach, the test approach and the economic approach being the other three. For details of the stochastic approach, see, for example, Aldrich (1992), Clements et al. (2006), Diewert (2007) and Selvanathan and Rao (1994).

(3.3) $\operatorname{var}\hat{\alpha}_e = \frac{\sigma^2}{n}, \quad \operatorname{var}\hat{\beta}_i = \frac{\sigma^2}{E}\left(1-\frac{1}{n}\right).$

The first expression states that the sampling variance of the estimate of the common peak is proportional to the variance of the disturbance term. Thus, it is more difficult to estimate this peak precisely when there is more dispersion among the peaks of the disaggregated variables.[4]

Equation (3.1) is a panel model with unevenly spaced observations. It should be noted that any pattern of serial correlation of the turning points cannot be directly implied by the serial properties of the underlying series that are being dated. Consider a monthly series p_{it}, with $t=1,\ldots,T$, and let S_{it} be a binary variable taking the value one if p_{it} is in an expansion phase at time t, and zero otherwise. Suppose time t is declared a peak of this series if and only if $S_{it}=1$ and $S_{i,t+1}=0$. With monthly observations, p_{it} is likely to be highly serially correlated, as is the binary series S_{it}. Define $y_{it}=S_{it}\left(1-S_{i,t+1}\right)t$, which equals date t at peaks and zero otherwise. The series y_{it} is serially correlated since most of time $y_{it}=0$. However, only the nonzero values of y_{it} are used as the dependent variable y_{ie} in model (3.1). While it is usual to set the length of the whole cycle, $y_{i,e+1}-y_{i,e}$, to be at least 1-2 years, the actual date of the subsequent turning point, $y_{i,e+1}$, is not really dependent on y_{ie}. The duration of the cycle $y_{i,e+1}-y_{i,e}$ varies as some recessions are more serious than others with a longer time being required to recover. For US business cycles dated by the NBER from 1854-2010, for example, there are 33 cycles with an average duration of about 56 months. In terms of phases, the shortest contraction is 6 months while the longest expansion is 120 months.[5] Therefore, given the date of the previous peak, it is almost impossible to anticipate the time of the following peak. Thus, there is likely to be little or no serial correlation of turning points y_{ie}.

As an exogenous shock may affect more than one market and as some markets are inherently more volatile than others, the individual series of interest could be correlated and have different volatilities. This implies that the covariance matrix $\mathbf{\Sigma}_e$ in model (3.1) is not necessarily scalar. So we could consider setting $\mathbf{\Sigma}_e=\sigma_e^2\mathbf{\Sigma}$, where $\mathbf{\Sigma}$ is a full rank symmetric

[4] As noted by Harding and Pagan (2006), Burns and Mitchell (1946, p. 70) in their influential work were aware of the dispersion of turning points: They observed that at any point in time "some activities [are] in an expanding phase, some beginning to recede from their peaks, some contracting, and some beginning to revive from their troughs", but "at any one time one phase is dominant".

[5] Source: http://www.nber.org/cycles/cyclesmain.html.

matrix, and σ_e is a scalar dealing with episodic heteroscedasticity. In what follows, to deal with this we use feasible generalised least squares (FGLS) and panel-corrected standard errors (Beck and Katz, 1995). In addition, our simulations investigate the consequences of taking $\mathbf{\Sigma}$ to be full or restricted as diagonal.

Stock and Watson (2010) use NBER business cycles as reference episodes. While this is a natural choice for their purpose, such natural reference cycles do not exist in many other instances. In such cases, a related aggregated series or the most important individual series could be used as a reference episode. In our application, we define episode e as $[t_e^* - \kappa, t_e^* + \kappa]$, where t_e^* is the e^{th} turning point of the reference series, and κ is the window width. For individual series i, if more than one turning point is dated, the peak (trough) with highest (lowest) value is selected as y_{ie}. If there is no peak (trough), then a missing value is recorded. Missing values typically occur for a variable that is very stable or exhibits mostly monotonic behaviour. Ignoring missing values can lead to a biased measure of the underlying cycle. For example, assume series i always peaks the last among n series and experiences no peak in episode e. One possible reason is that this series peaks at a point beyond the episode. Taking the average of all observed peaks, while ignoring the missing peak, gives an estimate that is likely to be earlier than the true underlying peak. Therefore, we use the conditional expectation from the expectation-maximization (EM) algorithm to replace missing values.

The estimated peak-to-peak duration of the cycle and its variance is

$$\hat{D}_e = \hat{\alpha}_e - \hat{\alpha}_{e-1}, \quad \mathrm{var}\left(\hat{D}_e\right) = \mathrm{var}\left(\hat{\alpha}_e\right) + \mathrm{var}\left(\hat{\alpha}_{e-1}\right) - 2\,\mathrm{cov}\left(\hat{\alpha}_e, \hat{\alpha}_{e-1}\right),\ e = 2,\ldots,E.$$

Here, duration is estimated as the difference between the estimates of the two peaks. Alternatively, one could first compute the durations of n disaggregated cycles, $\left(y_{1,e} - y_{1,e-1}\right),\ldots,\left(y_{n,e} - y_{n,e-1}\right)$, and then estimate the common duration as follows: Differencing equation (2.2) across episodes gives $y_{i,e} - y_{i,e-1} = \alpha_e - \alpha_{e-1} + \varepsilon_{i,e} - \varepsilon_{i,e-1} = D_e + \eta_e$, and the OLS estimator of D_e is exactly the same as $\hat{D}_e$ above. If $\varepsilon_{i,e}$ is serially independent, the duration disturbance η_e follows an MA(1) process and the approaches of Pagan and Nicholls (1976) and Ullah et al. (1986), for example, could be used.

4. Application to Metals

As an illustrative example, we use the monthly price data from 1989/06 to 2012/04 for six major non-ferrous metals: aluminium, copper, lead, nickel, tin and zinc. The prices are expressed in 2005 US dollars, deflated by the US Producer Price Index.[6] Let p_{it} be the price of metal i in month t and q_{it} be the corresponding volume. Then, $M_t = \sum_{i=1}^{6} p_{it}q_{it}$ is the total value and $w_{it} = p_{it}q_{it}/M_t$ is the value share of i.[7] If we write $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ for the log-change in the i^{th} price, then the Divisia price index is

$$DP_t = \sum_{i=1}^{6} \bar{w}_{it} Dp_{it}, \qquad (4.1)$$

where $\bar{w}_{it} = 1/2 \cdot \left(w_{it} + w_{i,t-1}\right)$ is the average value share over months t and t-1. This index weights prices according to the relative economic importance of the metals.

We use the Bry-Boschan (BB, 1971) algorithm to date the turning points in the price index as well as individual prices.[8] For convenience, we refer to the period from a peak to the next trough as a "slump" in prices and the subsequent recovery to the next peak as a "boom". Figure 4.1 gives the results in graphical form. The expansion that commenced in the early 2000s, known as the "Millennium Boom", was unusually long. The average duration of the phases is given in Table 4.1. Columns 3 and 5 show that even the average duration of phases differs substantially across metals, which points to uncertainty regarding the underlying cycle. The peaks and troughs of the price index are contained in columns 2 and 3 of Table 4.2. Figure 4.2 plots the price index with the slumps indicated. Two features of the index stand out: (i) Up to the year 2000 (month 128), the booms are shorter than the slumps. (ii) The index drops dramatically after the global financial crisis of 2007-2008 (after month 225, 2008/02), but then recovers almost fully within the following three years.

An important idea from index number theory is that goods should be weighted to reflect their economic importance, as in the price index 4.1. We apply this idea to the

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[6] The US Producer Price Index is from http://stats.oecd.org/Index.aspx?DataSetCode=REFSERIES. The metal prices are from Thompson Reuters Datastream and refer to the last trading day of the month. For prior studies on the cyclical behaviour of metal prices, see Cashin et al. (2002), Davutyan and Roberts (1994), Labys et al. (1998) and Roberts (2009).

[7] The turnover volume on the London Metals Exchange is used as the measure of q_{it}. To reduce the large amount of noise, turnover is smoothed using a 7-point unweighted centred moving average. Prices are not smoothed. For a discussion of this treatment, see Pagan and Sossounov (2003) and Cashin et al. (2002). The turnover data are from Thompson Reuters Datastream.

[8] We use Adrian Pagan's MATLAB program to implement the BB algorithm, available at http://www.ncer.edu.au/data/. The BB algorithm involves the following steps: (i) Identification of possible peaks (troughs) as local maxima (minima) using a window comprising the previous five and next five months. (ii) Censoring of the peaks and troughs via three rules: (a) peaks and troughs must alternate – when there are two consecutive peaks (troughs), the higher (lower) of the two is kept; (b) peaks and troughs in the last 6 months and first 6 months of the sample period are eliminated; and (c) a phase (that is, a boom or a slump) must last for at least 6 months, and a cycle (the combined period of the boom and slump) must last for at least 15 months. We do not use the amplitude threshold parameter. Harding and Pagan (2003) compare the Markov switching model with the BB algorithm, and find the latter more appealing on the grounds of transparency, robustness, simplicity and reliability.

estimation of the underlying cycle by using the value of production of each metal, which is proportional to the value share. As the peaks for the six metals do not necessarily occur at the same time, using the value share of metal i at its peak in episode e, $w_{i,y_{ie}}$, implies $\sum_{i=1}^{n} w_{i,y_{ie}} \neq 1$, where y_{ie} denotes the date of the peak for i in e. An alternative is to use the value shares at the same time point, such as at the mid-point of the episode or the peak of the reference episode. Preliminary computations confirmed that the estimates are not sensitive to this choice. Therefore, we use the values shares as at the peak in episode e, t_e, for the weights and multiply both sides of model (3.1) by the diagonal matrix $\text{diag}\left\{\sqrt{w_{1,t_e}},\ldots,\sqrt{w_{n,t_e}}\right\}$.

Before applying model (3.1), we need to define the episodes. We do this by using the turning points of the reference variables. Three alternative reference variables are used: (i) the Divisia price index (4.1); (ii) the copper price, the most important metal in terms of value share; and (iii) that first proposed by Harding and Pagan (HP, 2006). The HP approach is implemented as follows: For each period t, calculate the distances from the peaks of the individual series lying in the window $[t-\kappa, t+\kappa]$. If the median distance for t is no longer than half a month, point t is taken to be the underlying peak. A similar procedure is used for troughs. Peaks and troughs so identified can be regarded as those corresponding to the (implicit) HP reference variable. As the minimum cycle length is 15 months in the BB algorithm, a window width of $\kappa = 7$ months is used. A comparison of the three reference variables is shown in Figure 4.3.

What is the impact of using the different reference variables? Table 4.2, which contains the dates of four sets of turning points of the underlying cycle, addresses this question. The Divisia price index turning points are presented in columns 2 and 3. This is an example of the aggregate-then-date approach. These turning points are compared with those of the date-then-aggregate estimates shown in columns 4 to 6.[9] When the Divisia price index is used as the reference, the month numbers of the estimated turning points in column 4 are very close to those of the price index in column 3. Except for the first trough, the discrepancy is around one month, suggesting that these estimates are approximately unbiased. The standard errors in column 4 fall mainly within the fairly narrow range of 0.7-1.1 months.[10]

[9] The fractions of month numbers here and subsequently should be rounded to the nearest integer to be actual time points.

[10] The implied estimates of duration (the differences between consecutive turning points, not shown in the table) are reasonably close to the values implied by the turning points of the Divisia price index. The exceptions are the boom from event T1, denoting the first trough, to P2, the second peak, and the slump from P6 to T6. The T1 to P2 difference is mainly caused by the estimate of T1, which is about 4 months earlier than that of the price index. The highly-weighted copper price has a trough in month 24, leading to an estimated trough in month 26.

When the copper price is used as the reference (column 5), an additional peak and two additional troughs are identified.[11] Other than this, however, the results are similar to those with the Divisia price index. Finally, the HP reference yields slightly fewer estimated turning points (column 6).[12] These estimates are again close to the turning points in column 3, although the standard errors are higher than those of the other two approaches. The estimates of the phase parameters are given in panel C of Table 4.2. The estimates for aluminium and tin are positive for all three cases, implying their prices reach a peak or trough later than the underlying market, while the estimate for lead is negative, indicating a market leader. The three sets of estimates in Table 4.2 are obtained by FGLS with a diagonal covariance matrix because there are insufficient observations when the HP reference is used. The simulations in the following section investigate the consequences of this restriction.

To summarise the above results, it can be said that while the number of estimated turning points is in part determined by the way episodes are defined, the differing approaches yield turning points that are quite similar. Most turning points are precisely estimated and close to those of the price index. Although in many instances the standard errors are relatively large, the estimated phase parameters for leads and lags are, for the most part, not very sensitive to alternative definitions of episodes. Thus, like Stock and Watson (2010), we find the date-then-aggregate approach to be viable and useful.

5. Simulations

The turning point y_{ie} for the price of metal i in episode e is obtained by applying the Bry-Boschan (BB) algorithm and is then used to estimate the underlying cycle with model (3.1). Accordingly, there are two types of uncertainties involved in the estimates: "model uncertainty" in the form of the disturbance term in equation (3.1); and "data uncertainty", referring to the BB algorithm identifying different turning points for data sets that differ from one instance to another because of random factors. To shed light on the reliability of the

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[11] The additional trough at the start of the period is not identified by the price index because the index data starts 4 months later than the individual prices. The reason is that the value shares (the weights in the index) start 4 months later as a moving average is used to smooth the volume data. It is interesting to note that the estimated additional peak (month 205, June 2006) and trough (month 212, January 2007) both have relatively large standard errors. This is due—at least in part—to a drop in the copper price by more than 20% in the second half of 2006 while the other metals experienced no similar price decreases around this time (Fig. 4.1).

[12] The peak in month 96 (May 1997) is not identified by this method as for this month the median distance from the peaks near the individual series is 1 month, while the other peaks have median distances no greater than 0.5 months. This raises the unresolved issue of how close to zero the median distance should be. Another issue is that the individual series are not equally weighted in this approach. The weight for a given series is not proportional to its economic size, but to its number of turning points. Thus, for example, a volatile series with more frequent cycles would be given higher weight.

approach, we investigate these two sources. For simplicity, episodes are defined by the Divisia price index.

Model uncertainty. In the first set of simulations, the disturbance vector $\boldsymbol{\varepsilon}_e$ in (3.1), $\mathbf{y}_e = \alpha_e \iota + \boldsymbol{\beta} + \boldsymbol{\varepsilon}_e$, is sampled from $N(\mathbf{0}, \sigma_e \boldsymbol{\Sigma})$, where $\sigma_e \boldsymbol{\Sigma}$ is the data-based estimate of the covariance matrix. For trial s, the sampled vector $\boldsymbol{\varepsilon}_e^{(s)}$ is added to the vector of fitted values, $\hat{\alpha}_e \iota + \hat{\boldsymbol{\beta}}$, to yield $\mathbf{y}_e^{(s)}$. The new turning points $\mathbf{y}_e^{(s)}$ $(e = 1, \ldots, 14)$ are then used to reestimate the parameters, denoted by $\hat{\alpha}_e^{(s)}$ and $\hat{\boldsymbol{\beta}}^{(s)}$. Table 5.1 summarises the results for $\hat{\alpha}_e^{(s)}$ over 10^4 trials. Column 1 presents the data-based FGLS estimates with panel-corrected standard errors that are used as the "true values". Panel A refers to the case when $\boldsymbol{\Sigma}$ in the data-generating process (the true value) is diagonal (the true values of column 1 in this panel are also based on a diagonal covariance matrix). In panel B, the true $\boldsymbol{\Sigma}$ is full. To examine the consequences of a false assumption regarding the form of $\boldsymbol{\Sigma}$, two simulations are presented in each panel. For trial s, $\hat{\alpha}_e^{(s)}$ and $\hat{\boldsymbol{\beta}}^{(s)}$ are estimated by FGLS first with a diagonal covariance matrix and then with a full covariance matrix, as indicated by the headings "Estimated $\boldsymbol{\Sigma} = \text{diagonal}$" and "Estimated $\boldsymbol{\Sigma}$ = full".

Consider the case in which the true and estimated covariance matrices are both diagonal (see columns 2-5 of panel A, Table 5.1). The means of the estimated turning points in column 2 are very close to the true values in column 1. However, the standard errors are somewhat low as the root-mean-squared standard errors (RMSSEs) of column 4 are on average about 25% below the root-mean squared errors (RMSEs, computed around the true values) of column 3. More or less the same picture emerges from the same columns of panel B, where the estimated covariance matrix continues to be assumed to be diagonal, but the true value is now full. Columns 6-9, however, reveal a major problem when the estimated covariance matrix is taken to be full: In column 9, the standard errors are now on average about 70% below the RMSEs. This result holds whether the true covariance matrix is diagonal or full. For this reason, we recommend a diagonal covariance matrix be used in applications.

Data uncertainty. To analyse the second source of uncertainty, we simulate data with the same number of individual series as before, $n = 6$, but now use more time-series observations with $T = 12 \times 50 = 600$ months.[13] A first-order VAR model is employed to

[13] While increasing n would help with asymptotic properties, the size of the covariance matrix would grow quadratically. Stock and Watson (2010) applied stratified random sampling to control the number of individual variables involved.

generate the prices. Given a price vector at time t, $\mathbf{p}_t = [p_{1t}, \ldots, p_{6t}]'$, the log of prices at time t+1 is $\log \mathbf{p}_{t+1} = \mathbf{A} \cdot \log \mathbf{p}_t + \mathbf{c} + \boldsymbol{\mu}_{t+1}$, where $\mathbf{A}$ is a 6×6 matrix of coefficients, $\mathbf{c}$ is an intercept vector and the error term $\boldsymbol{\mu}_{t+1}$ is $N(\mathbf{0}, \boldsymbol{\Sigma}_{\mu})$. For trial s, we generate $\log \mathbf{p}_{t+1}^{(s)},\ t = 1,\ldots,599,$ with the prices observed in the first month used for $\log \mathbf{p}_1$. We use the BB algorithm to date the individual series, the Divisia price index as the reference cycle and the EM algorithm for missing observations, and then re-estimate model (3.1) using FGLS with a diagonal covariance matrix, as before. The estimated turning points, $\hat{\alpha}_e^{(s)}$, can then be compared with those of the derived Divisia price index, denoted by $\theta_e^{(s)}$.[14] Figure 5.1 gives the differences $\hat{\alpha}_e^{(s)} - \theta_e^{(s)}$ from a representative simulation for e = 1,...,33 episodes. The differences are modest with a mean of -0.02 months. With the exception of trough 6 and peak 13, the estimates are within a three-month band.

As each trial contains a different number of episodes, to make trials comparable we use the mean and RMSE over episodes, $\sum_{e=1}^{E^{(s)}} \left(\hat{\alpha}_e^{(s)} - \theta_e^{(s)}\right) \big/ E^{(s)}$ and $\sqrt{\sum_{e=1}^{E^{(s)}} \left(\hat{\alpha}_e^{(s)} - \theta_e^{(s)}\right)^2 \big/ E^{(s)}}$, repectively, where $E^{(s)}$ is the number of episodes in trial s. Panel A of Figure 5.2 shows that the mean discrepancy in 10,000 trials is about 0.11 months, which is obviously small. However, as the standard deviation here is much larger at 0.39 months, there is considerable dispersion with some estimated turning points occurring some time earlier than those of the price index, and some much later. Panel B shows that the RMSE mean is about 1.8 months and the standard deviation 2.5.[15]

In summary, the average turning point discrepancies are small for both sources of uncertainty. However, there is still considerable uncertainty in the estimates of the underlying turning points: the average RMSE is about 0.7 months for the model uncertainty case, while it is 1.8 months for data uncertainty. Thus, it seems that randomness in the dating of the turning points of the individual series is more of an issue than model uncertainty.

6. Indexes and Cycles

The stochastic approach to index numbers treats the rate of inflation as an unknown parameter to be estimated as the common trend in n noisy prices, while allowing for

[14] Note to referees: Further details of the simulation procedure are given in the attached Supplement.

[15] In about 0.5% of the trials, the EM algorithm failed to converge after 500 iterations, so we used the values at that point. If those trials are discarded, the mean and standard deviation of the RMSEs drop to 1.6 and 0.4 months, respectively.

differences in individual prices due to relative price changes. This approach enables inferences to be made regarding the true rate of inflation. As shown above, the stochastic approach can be applied to the dates of the turning points of n disaggregated series, with the estimated peak of the underlying cycle a weighted average of n individual peaks after adjusting for systematic leads/lags of the disaggregated series. The link between index number theory and the cycle could open up new possibilities. For example, one branch of index numbers examines the moments of the distributions of n price and quantity changes in the form of Divisia price indexes (the first moments), variances and the price-quantity covariance (Theil, 1967, Chap. 5). A similar approach could be used to analyse the distributions of duration and amplitude-related information.

As an illustrative example, rather than simply dating the turning points of the underlying cycle, we consider the status at each time point t to determine how far the cycle is from the nearest turning point. To do this, define, for series i, D_{it}^{P} as the time distance between t and the closest peak within the cycle for this variable. For example, suppose series i, the price of good i, runs from t = 0,...,T and has three consecutive turning points, a peak at t_{i1}, a trough at t_{i2} and a peak at t_{i3}, as shown in panel A of Figure 6.1. Panel B contains the distance measure, $D_{it}^{P} = t - t_{i1},\ t \in (0, t_{i2}]$ and $D_{it}^{P} = t - t_{i3},\ t \in (t_{i2}, T]$. These are 45-degree lines for $(0, t_{i2}]$ and $(t_{i2}, T]$. At any time t in a boom, when the price moves from a trough towards a peak, such as period $(0, t_{i1})$ or (t_{i2}, t_{i3}), $D_{it}^{P} < 0$ as the peak occurs after t and $D_{it}^{P} > 0$ in slumps. At the peaks t_{i1} and t_{i3}, $D_{it}^{P} = 0$. In general, the peak relevant for distance in a trough-to-trough cycle is defined as the closest peak to any t within the cycle. Clearly, a discontinuity occurs at trough t_{i2} as before then the closest peak is t_{i1}, and t_{i3} thereafter. For incomplete cycles, for example, if there were no peak before t_{i2}, $D_{it}^{P} = t - t_{i3},\ t \in (0, T]$. Panel C of Figure 6.1 contains the values of the distance measures.

If there are i=1,...,n series, how can the individual distances from the peak be combined to give the distance from the closest overall peak, D_{t}^{P}? As before, if series i consistently lags the overall sector by β_i months, then its "corrected" distance from the common peak is $D_{it}^{P} - \beta_i$, which now provides a noisy observation on D_{t}^{P}. Thus, we have the following distance model:

(6.1) $\quad D_{it}^{P} - \beta_i = D_{t}^{P} + \varepsilon_{it}, \quad$ i=1,…,n, t=1,…,T, where ε_{it} is a disturbance term. Taking into account the relative importance of different markets, as in index number theory, one simple estimator of D_t^P is $\sum_{i=1}^n w_{it}\left(D_{it}^P - \beta_i\right) = \sum_{i=1}^n w_{it} D_{it}^P$, if $\sum_{i=1}^n w_{it}\beta_i = 0$, where w_{it} is the value share of series i at time t. When $\hat{D}_t^P < 0$, the sector as a whole is between a trough and a peak at time t, so it could be said that things are "improving"; the converse applies when $\hat{D}_t^P > 0$, as the peak has been passed; and when $\hat{D}_t^P = 0$, time t is an overall peak. This weighted average approach gives the centre of gravity of the distribution of distance, while the corresponding weighted variance

$$V_t^P = \sum_{i=1}^n w_{it}\left(D_{it}^P - D_t^P\right)^2$$

measures dispersion around the overall distance. This variance varies with time and is larger when there is greater dispersion of cyclical status among the individual series; if all series peak at the same time, the variance is zero.

Panel A of Figure 6.2 plots the average distance from the peaks and the one-standard deviation band, $\left[\hat{D}_t^P - \sqrt{V_t^P},\ \hat{D}_t^P + \sqrt{V_t^P}\right]$, based on the n=6 metal prices. The potential peaks are located by the value of $\hat{D}_t^P$ crossing the zero horizontal line. This occurs seven times in the figure. As can be seen, each is reasonably close to the corresponding peak of the Divisia price index from Section 4 (indicated by red circles), implying some support for this approach. Although the large falls in $\hat{D}_t^P$ occur at troughs, the locations of the troughs are not as obvious as the peaks since the fall depends on the durations of the past slump and following boom. Alternatively, we can define D_{it}^T as the distance from the closest trough for series i. Proceeding as before, time t is a trough if $\hat{D}_t^T = \sum_{i=1}^n w_{it} D_{it}^T = 0$. Panel B of Figure 6.2 plots $\hat{D}_t^T$ and the one-standard deviation band. Here, the distance measure hits zero seven times also, but the first hit, in 1990/01, is an extra potential trough.[16] The last trough of the Divisia price index of 2011/09 is not detected since $\hat{D}_t^T$ is far above zero in 2011.[17]

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[16] The reasons for this are the same as those discussed in footnote 11.

[17] The last turning point is associated with peaks for aluminium, lead and tin, but troughs for copper, nickel and zinc. That is, during the last few months of the sample period, there are slumps for aluminium, lead and tin, but booms for copper, nickel and zinc. Usually, during a slump (boom) the distance from the closest trough is negative (positive). But because of the incomplete cycles during this period for aluminium, lead and tin, the distances for these metals are positive. For copper, nickel and zinc (no incomplete cycles), the distances are positive as they are booming, leading to the overall distance to also be positive.

The band width in Figure 6.2 indicates the degree of dispersion of co-movement. In panel B, the band is especially wide during 2006-2007. At least part of this disparate behaviour is due to the price of copper (the most important metal). There was a short slump in this price in late 2006, a trough being detected by the BB algorithm in 2007/01, while the prices of the other metals continued to increase as part of the Millennium Boom. Table 6.1 compares the turning points dated by $\hat{D}_t^P = 0$ and $\hat{D}_t^T = 0$ with those of the Divisia price index. Column 7 shows that the agreement is not perfect, but is reasonable as the differences are mostly of the order of a couple of months. The main exception is the 6-month difference for the trough before the Millennium Boom. Thus, the approach seems to provide reasonable dates of turning points and has the advantage of not requiring any prior grouping of events into episodes. However, if a general idea of the nature of the episodes is available, then the panel model of Section 3, model (3.1), is probably the desirable way to model the underlying cycle.[18]

This approach can also be applied to prices to locate the overall turning point, which could be described as the "vertical dimension" of the problem (time being the horizontal). The basic ingredient is now the difference between the log price at time t and that at the closest peak or trough for each series. The point with zero overall price difference would then be the turning point of the underlying cycle. Figure 6.3 is constructed in exactly the same manner as Figure 6.2, but price differences now replace time differences. As each price is always lower than its closest peak and higher than its closest trough, the (weighted) average price difference is always negative for the peaks in panel A and positive for the troughs in panel B. Thus, the local maxima in panel A are potential peaks and the local minima in panel B are potential troughs. These are reasonably close to the turning points of the Divisia price index (indicated by red circles), as before. An extension would be to combine the time and price differences to simultaneously determine the overall turning point. As this would involve considerably more data, the problem of missing observations and small samples would be reduced.

7. Concluding Comments

A basic measurement problem is to determine the current state of an economy or a sector – is the economy expanding or contracting, or is the general level of prices increasing

[18] As discussed before, Harding and Pagan (2006) also use a measure of distance from turning points in a multivariate context. For each series and each period, they consider the distance from the respective turning points; period t is identified as a turning point for the overall market if the median distance is zero. The difference between their approach and the above is that we consider the closest turning points and allow for the relative importance of each series in measuring distance from the overall turning point.

or decreasing? In a data-rich environment with multiple indicators of current economic conditions, the question is how to combine potentially conflicting indicators into one measure. In a recent paper, Stock and Watson (2010) introduced a new approach to measuring the business cycle in a multiple indicator context. They start with n indicators of the state of the macroeconomy and date their turning points using conventional methods, so there are n peaks or n troughs in each episode. These dates are viewed as reflecting the combined influences of the behaviour of the underlying cycle, factors specific to individual indicators, and random events. Stock and Watson then aggregate the individual dates into peaks and troughs of the underlying cycle using a panel regression procedure with time and indicator effects. The time effects represent the underlying cycle, while the indictor effects measure the leads or lags of the individual turning points relative to the underlying one. This procedure has the advantage of recasting cycle measurement into an econometric framework to provide *estimates* of the dates of the turning points with standard errors measuring estimation uncertainty. These standard errors, reflecting the dispersion of the turning points among individual series, are new to the cycle measurement literature and have wide applicability. This rich approach could be used to study other aspects of the cycle such as amplitude, duration and asymmetries.

A seemingly different line of research is the stochastic approach to index numbers, whereby there are n individual prices that are regarded as noisy readings on the underlying rate of inflation. The problem is to extract the common signal by combining the prices to minimise the noise. This leads to an estimate of the underlying rate of inflation and the systematic changes in relative prices (see, e. g., Clements et al., 2006, and Selvanathan and Rao, 1994). In this paper, we showed how Stock and Watson's approach can be considered as a stochastic index number problem. In this context, we showed how this approach can be extended and enhanced. We allowed the individual indicators to be unequally weighted to reflect their differing economic importance. We showed how the approach can be used to characterise the whole path of the underlying cycle, rather than just the turning-point dates. The viability of this approach to dating cycles was illustrated with world metal prices.

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TABLE 2.1

PEAKS OF THREE VARIABLES

(Month number)

Disaggregated variable	Scenario		
	1	2	3
(1)	(2)	(3)	(4)
1	5	3	1
2	5	5	5
3	5	7	9
Mean	5	5	5
Standard deviation	0	$\sqrt{8/(3-1)}=2$	$\sqrt{32/(3-1)}=4$
Standard error of mean	0	$2/\sqrt{3}=1.2$	$4/\sqrt{3}=2.4$

TABLE 4.1

SUMMARY OF PHASES IN METAL PRICE CYCLES

Metal	Slumps		Booms	
	No. of episodes	Duration (No. of months)	No. of episodes	Duration (No. of months)
(1)	(2)	(3)	(4)	(5)
Aluminium	7	19	7	20
Copper	8	13	9	20
Lead	9	15	9	15
Nickel	6	26	7	18
Tin	5	30	5	22
Zinc	6	20	7	18
Mean				
All	6.83	21	7.33	19
No MB	5.33	22	5.66	12

Notes: Columns 3 and 5 are averages. The mean of the last row excludes the atypically long Millennium Boom (MB).

TABLE 4.2

THREE SETS OF ESTIMATES OF THE CYCLE MODEL

Price index			Estimates of turning points with episodes defined by (Month number)		
Event	Date	Month number	Price index	Copper price	Harding-Pagan
(1)	(2)	(3)	(4)	(5)	(6)
			A. Peaks		
P1	1990/08	15	14.73 (0.83)	15.23 (0.79)	15.18 (1.26)
P2	1992/07	38	36.94 (0.70)	37.17 (0.71)	37.11 (1.04)
P3	1995/01	68	67.12 (0.72)	67.39 (0.72)	67.31 (1.09)
P4	1997/05	96	97.05 (0.80)	97.00 (0.83)	
P5	2000/01	128	127.63 (0.81)	135.44 (1.04)	127.56 (1.21)
				205.32 (0.93)	
P6	2008/02	225	228.13 (0.80)	227.30 (0.79)	226.84 (1.19)
P7	2011/02	261	261.33 (0.73)	261.58 (0.69)	261.52 (1.07)
			B. Troughs		
				7.92 (0.78)	7.90 (1.23)
T1	1991/12	31	26.33 (0.75)	25.92 (0.81)	26.01 (1.17)
T2	1993/11	54	52.73 (0.70)	52.91 (0.70)	52.96 (1.10)
T3	1996/09	88	88.3 (0.83)	87.86 (0.83)	
T4	1999/01	116	117.23 (0.70)	117.19 (0.73)	117.25 (1.05)
T5	2001/10	149	148.57 (1.08)	149.16 (0.98)	154.90 (0.98)
				212.06 (1.17)	
T6	2009/01	236	235.93 (0.65)	236.08 (0.67)	235.97 (0.96)
T7	2011/09	268	269.16 (1.07)	268.41 (0.98)	
			C. Phase Leads/Lags (Months)		
Aluminium			0.50 (0.41)	0.51 (0.44)	1.09 (0.59)
Copper			-0.09 (0.40)	-0.13 (0.36)	-0.31 (0.61)
Lead			-1.50 (0.85)	-0.59 (0.77)	-0.83 (0.90)
Nickel			0.08 (0.64)	-1.10 (0.72)	-0.59 (0.90)
Tin			1.27 (0.73)	0.62 (0.68)	0.38 (1.05)
Zinc			-0.25 (0.55)	0.69 (0.75)	0.25 (0.85)

Notes: Pi and Tj represent the i^{th} peak and j^{th} trough, respectively. Standard errors are in parentheses.

TABLE 5.1

SIMULATION RESULTS FOR TURNING POINT ESTIMATES

	True value of turning point	Estimated ∑ = diagonal				Estimated ∑ = full			
		Mean	RMSE	RMSSE	(4)/(3)	Mean	RMSE	RMSSE	(8)/(7)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				A. True Σ = diagonal					
P1	14.73 (0.75)	14.74	0.76	0.55	0.72	14.74	0.73	0.24	0.33
T1	26.33 (0.74)	26.34	0.74	0.55	0.74	26.33	0.73	0.23	0.32
P2	36.94 (0.70)	36.94	0.71	0.54	0.76	36.94	0.70	0.21	0.30
T2	52.73 (0.73)	52.74	0.74	0.54	0.74	52.74	0.72	0.22	0.31
P3	67.12 (0.73)	67.11	0.74	0.55	0.73	67.12	0.72	0.23	0.31
T3	88.30 (0.72)	88.30	0.73	0.54	0.74	88.30	0.71	0.22	0.30
P4	97.05 (0.69)	97.05	0.70	0.54	0.78	97.05	0.71	0.22	0.31
T4	117.23 (0.67)	117.24	0.66	0.53	0.80	117.23	0.69	0.21	0.31
P5	127.63 (0.67)	127.63	0.67	0.54	0.81	127.63	0.70	0.24	0.34
T5	148.57 (0.69)	148.57	0.69	0.54	0.77	148.57	0.70	0.22	0.31
P6	228.13 (0.70)	228.12	0.70	0.54	0.77	228.14	0.71	0.20	0.29
T6	235.93 (0.69)	235.93	0.69	0.53	0.77	235.93	0.71	0.20	0.29
P7	261.33 (0.72)	261.33	0.73	0.54	0.74	261.33	0.72	0.22	0.31
T7	269.16 (0.72)	269.16	0.74	0.54	0.73	269.17	0.72	0.22	0.30
	Mean		0.71	0.54	0.76		0.71	0.22	0.31
				B. True Σ = full					
P1	14.77 (0.36)	14.76	0.44	0.32	0.74	14.77	0.43	0.15	0.35
T1	26.37 (0.33)	26.37	0.42	0.32	0.76	26.37	0.42	0.15	0.35
P2	36.95 (0.30)	36.95	0.41	0.31	0.77	36.95	0.41	0.14	0.34
T2	52.75 (0.31)	52.75	0.42	0.32	0.75	52.76	0.42	0.14	0.34
P3	67.14 (0.31)	67.15	0.44	0.33	0.74	67.13	0.42	0.15	0.34
T3	88.09 (0.30)	88.08	0.42	0.32	0.75	88.09	0.42	0.14	0.34
P4	97.04 (0.28)	97.04	0.42	0.33	0.77	97.04	0.42	0.15	0.36
T4	117.22 (0.31)	117.21	0.42	0.32	0.77	117.21	0.41	0.14	0.34
P5	127.43 (0.32)	127.42	0.43	0.33	0.76	127.43	0.43	0.17	0.39
T5	148.50 (0.30)	148.50	0.43	0.33	0.77	148.51	0.43	0.15	0.34
P6	228.35 (0.29)	228.35	0.42	0.32	0.77	228.35	0.41	0.14	0.33
T6	235.92 (0.30)	235.93	0.41	0.32	0.77	235.93	0.42	0.13	0.32
P7	261.34 (0.31)	261.34	0.42	0.31	0.75	261.34	0.42	0.14	0.34
T7	269.16 (0.31)	269.16	0.42	0.32	0.76	269.16	0.42	0.14	0.33
	Mean		0.42	0.32	0.76		0.42	0.14	0.35

Note: Panel corrected standard errors are in parentheses.

TABLE 6.1

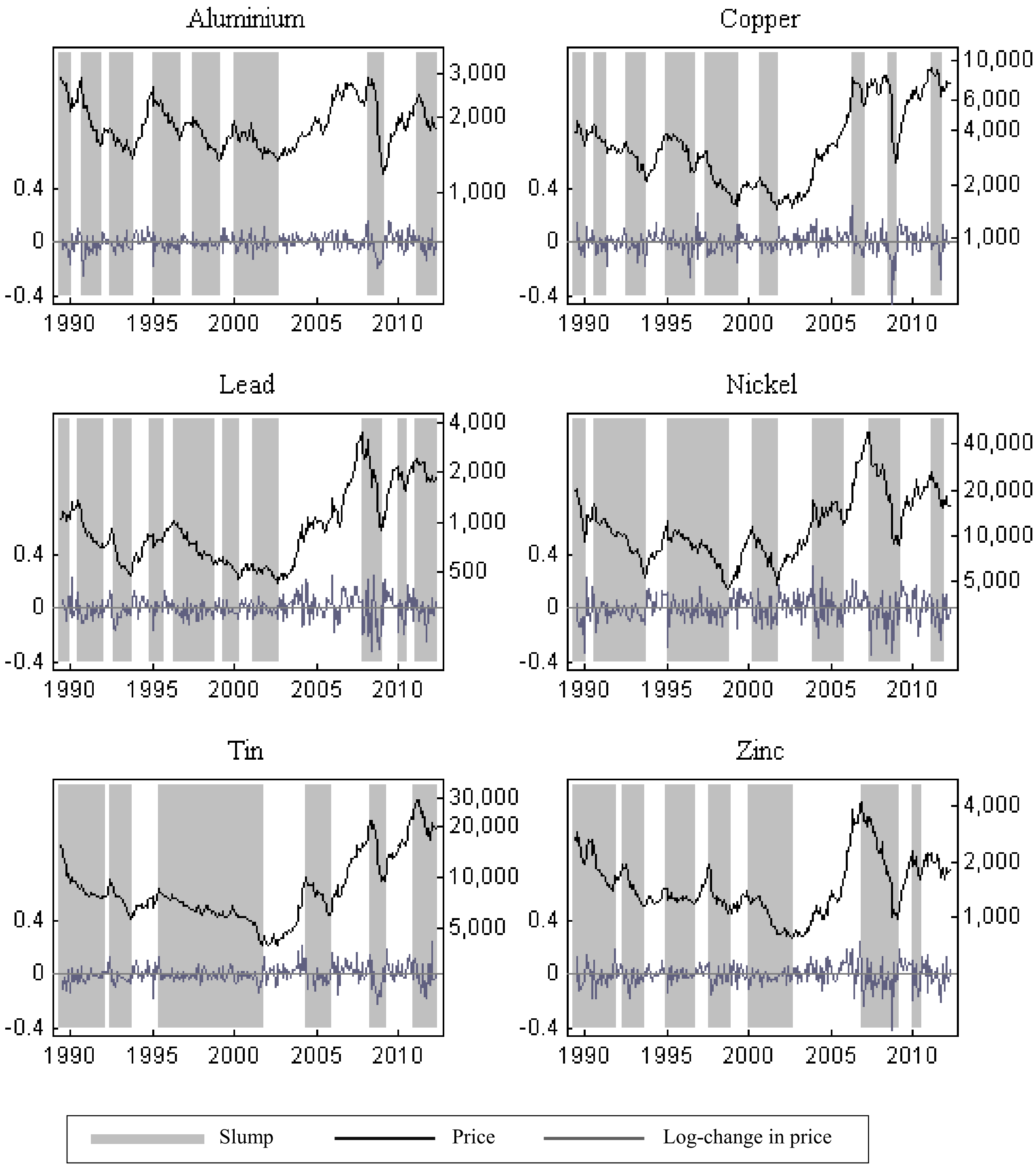
TWO SETS OF TURNING POINTS

	Divisia price index		$D_t^T \approx 0$ or $D_t^P \approx 0$			Error
Event	Date	Duration	Date	Duration	SD	(4)-(2)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
P1	1990/08		1990/10		5.92	2
T1	1991/12	16	1991/08	12	7.42	-3
P2	1992/07	7	1992/05	9	5.26	-2
T2	1993/11	16	1993/10	17	3.78	-1
P3	1995/01	14	1994/12	14	0.84	-1
T3	1996/09	20	1997/01	25	10.78	4
P4	1997/05	8	1997/02	1	9.84	-3
T4	1999/01	20	1999/03	23	5.31	2
P5	2000/01	12	2000/02	11	9.12	1
T5	2001/10	21	2002/04	26	6.79	6
P6	2008/02	76	2008/01	69	5.99	-1
T6	2009/01	11	2009/01	12	3.40	0
P7	2011/02	25	2011/01	24	3.93	-1
T7	2011/09	7				

Notes: Column 4 shows the dates when the distance is closest to zero. The standard deviation (SD) in column 6 is the SD of D_{it}^P or D_{it}^T over $i = 1,\ldots,6$.

FIGURE 4.1

METAL PRICE CYCLES



Note: The black lines are the prices, which refer to the right-hand axes (in 2005 US dollars per tonne, log scale). The grey lines hovering around zero are the monthly price log-changes, which refer to the left-hand axes. Shaded areas are the peak-to-trough slumps.

FIGURE 4.2

METAL PRICE INDEX CYCLES

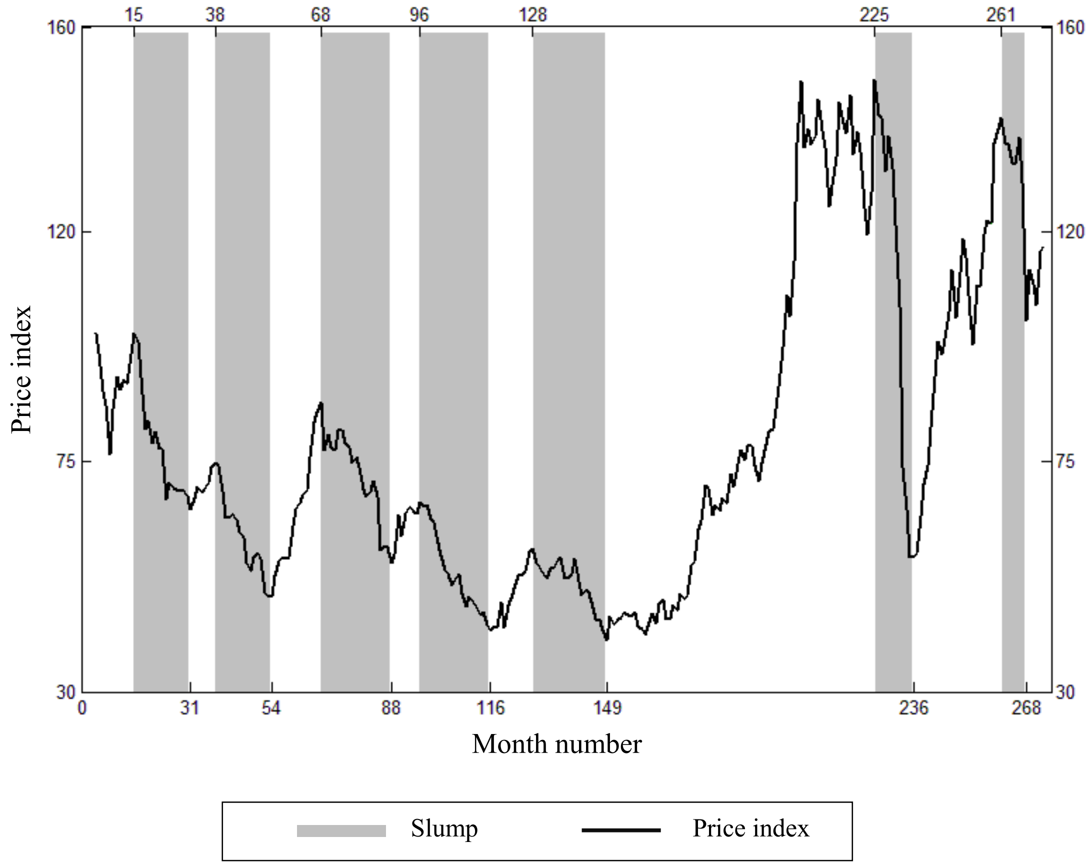
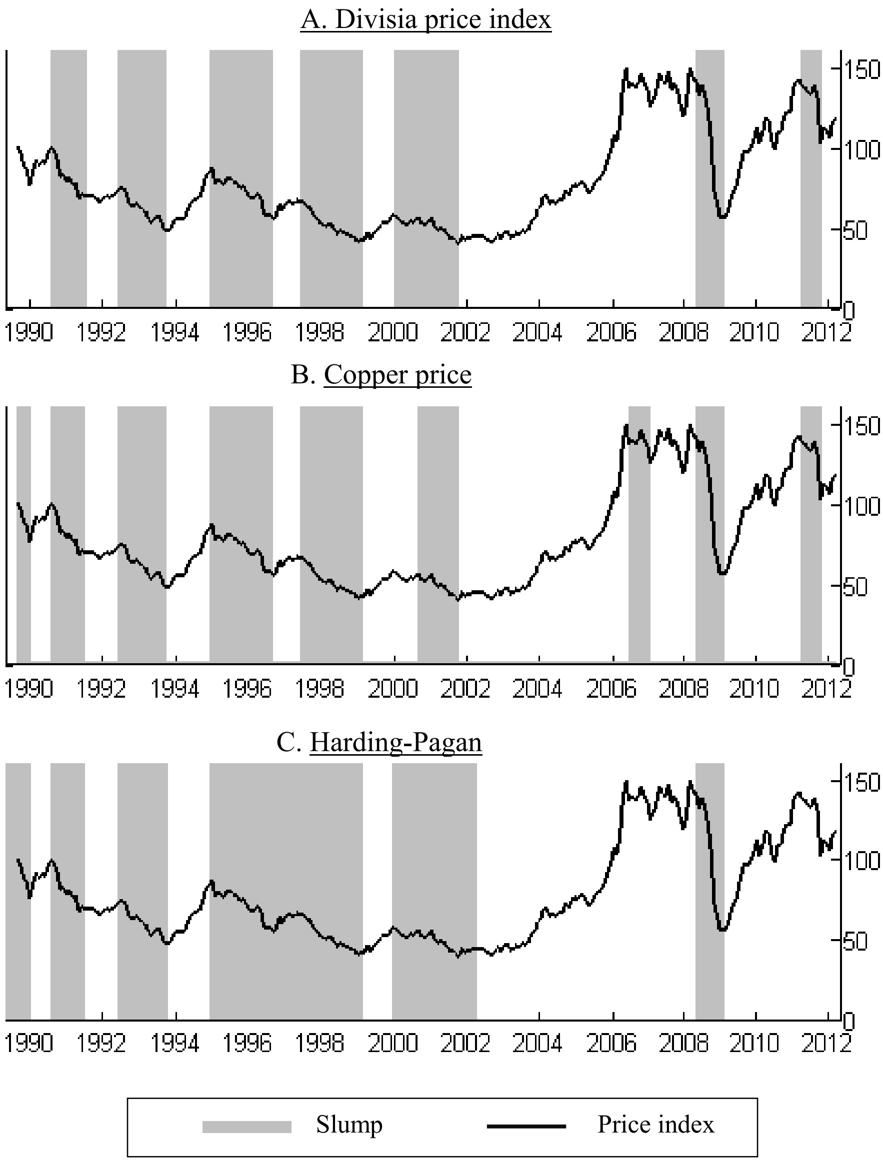


FIGURE 4.3

THE THREE REFERENCE VARIABLES



Note: The series plotted in each panel is the Divisia price index.

FIGURE 5.1

TURNING POINT DISCREPENCIES, REPRESENTATIVE TRIAL

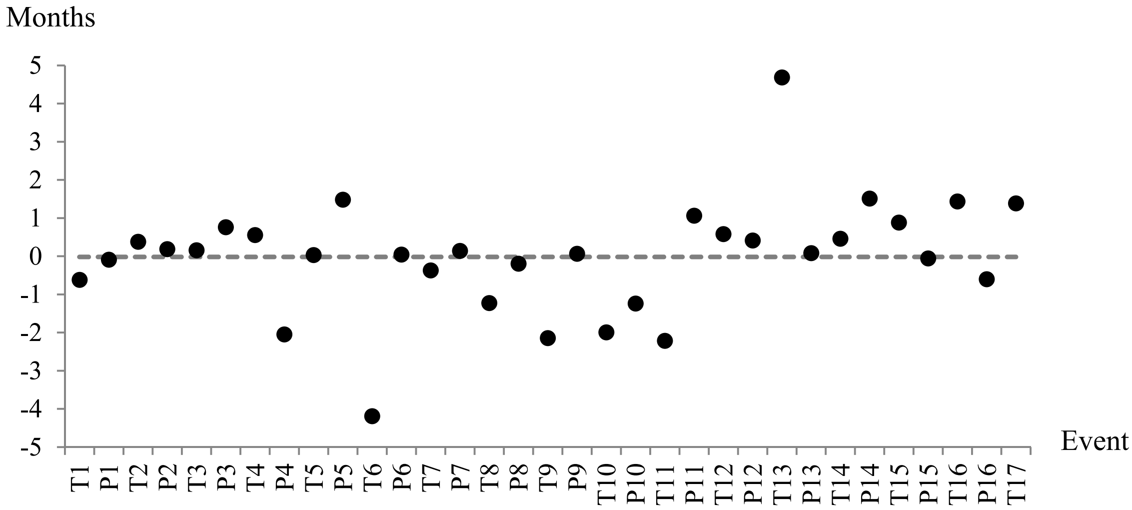
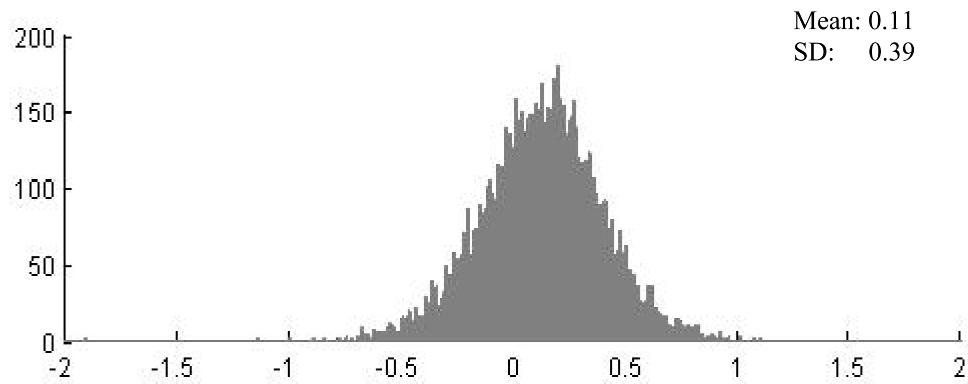


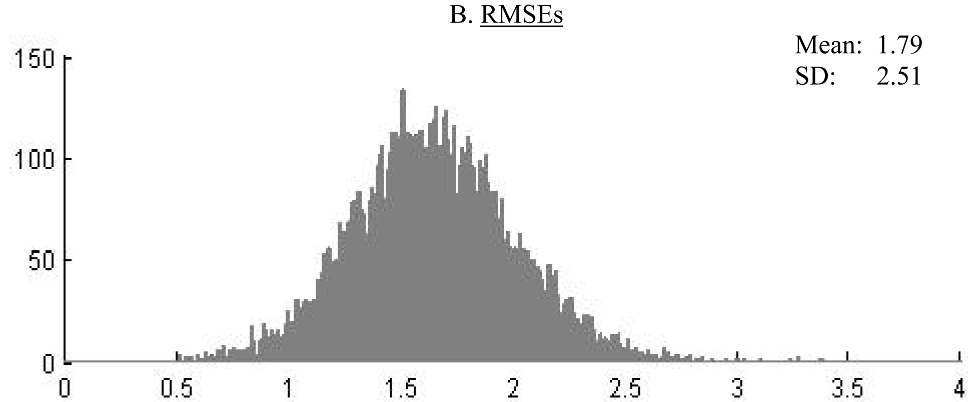
FIGURE 5.2

SIMULATED TURNING POINT DISCREPANCIES
(Months)

A. Means



B. RMSEs



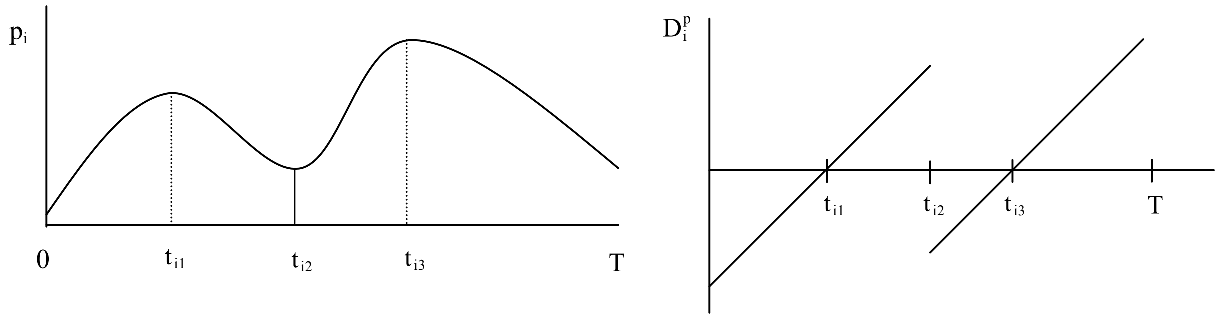
Note: In about 0.5% of 10,000 trials the root-mean squared errors (RMSEs) are greater than 4 months. To improve visualisation, these are truncated in panel B but are included in the mean and standard deviation (SD) figures shown.

FIGURE 6.1

TURNING POINTS AND DISTANCE

A. Price of good i

B. Distance from closest peak



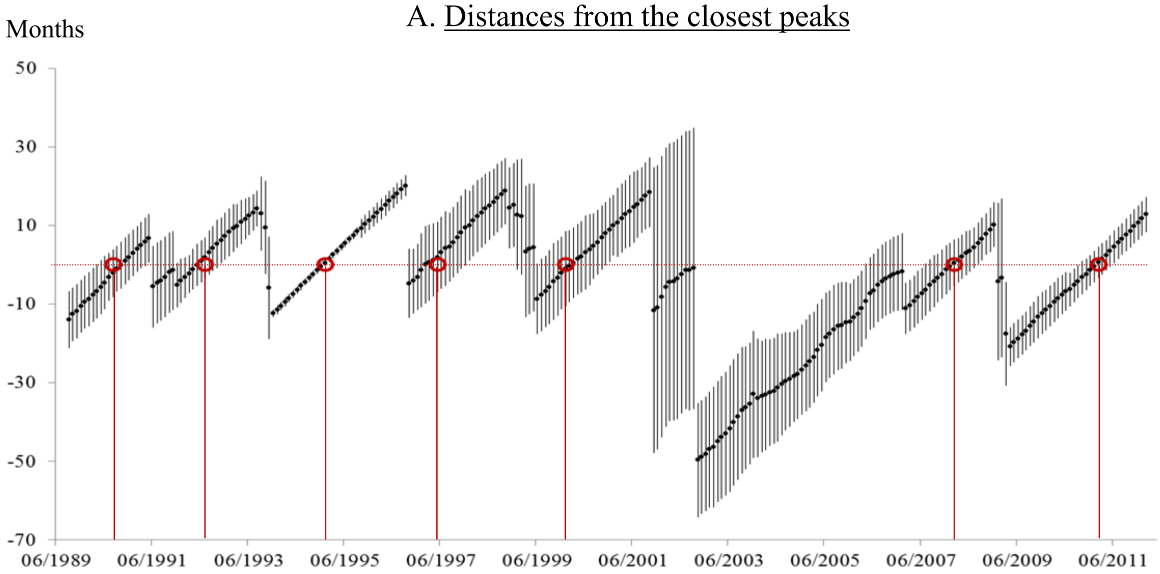
C. The Timeline

t:	1	…	$t_{i1}-1$	t_{i1}	$t_{i1}+1$	…	$t_{i2}-1$	t_{i2}
D_{it}^P:	$1-t_{i1}$	…	-1	0	1	…	$t_{i2}-1-t_{i1}$	$t_{i2}-t_{i1}$
t:	$t_{i2}+1$	…	$t_{i3}-1$	t_{i3}	$t_{i3}+1$	…	T	$t_{i2}+1$
D_{it}^P:	$t_{i2}+1-t_{i3}$	…	-1	0	1	…	$T-t_{i3}$	$t_{i2}+1-t_{i3}$

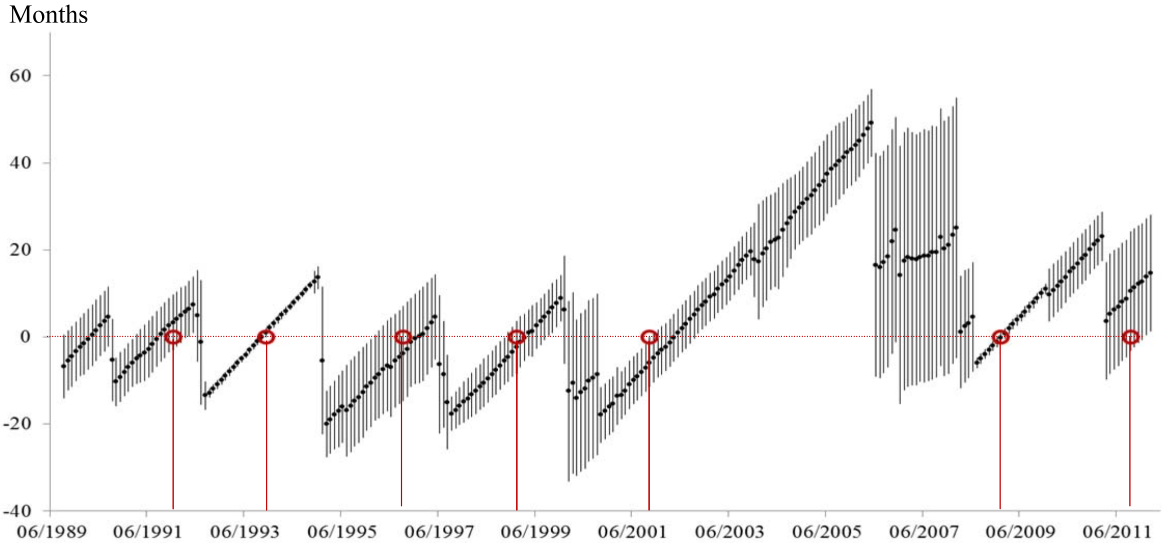
FIGURE 6.2

DISTANCES FROM THE CLOSEST TURNING POINTS

A. Distances from the closest peaks



B. Distances from the closest troughs

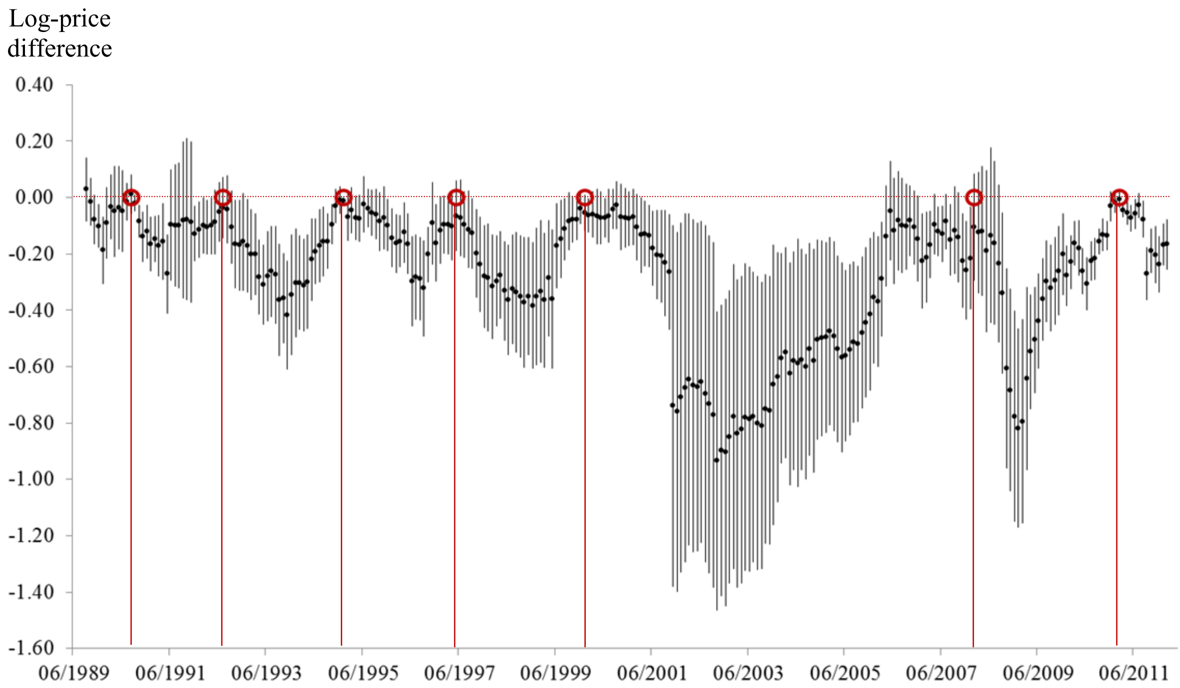


Note: Each dot represents the value of overall time distance from the nearest turning point. The length of each vertical line represents dispersion, defined as ± 1 standard deviation, centred on the corresponding dot. The small red circles indicate turning point dates of the Divisia price index from Section 4.

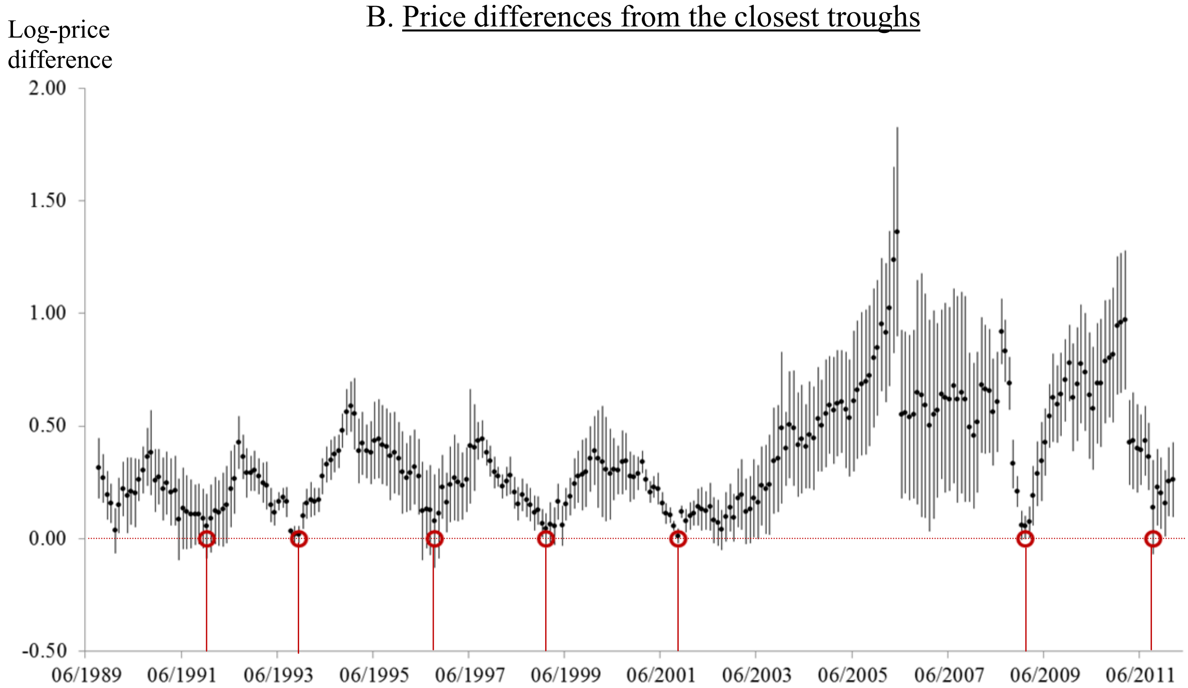
FIGURE 6.3

PRICE DIFFERENCES FROM THE CLOSEST TURNING POINTS

A. Price differences from the closest peaks



B. Price differences from the closest troughs



Note: Each dot represents the value of overall price difference from the nearest turning point. The length of each vertical line represents dispersion, defined as ± 1 standard deviation, centred on the corresponding dot. The small red circles indicate turning point dates of the Divisia price index from Section 4.

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Supplement

SIMULATING PRICES AND TURNING POINTS

This supplement describes in detail the simulation for the "Data uncertainty" part of Section 5.

Define the vector of the individual prices at time t as $\log \mathbf{p}_t = [\log p_{1t},\ldots,\log p_{6t}]'$. The prices at t+1 are generated by a VAR model:

$$\log \mathbf{p}_{t+1} = \mathbf{A}\cdot\log\mathbf{p}_t + \mathbf{c} + \boldsymbol{\mu}_t,\ \ t=1,\ldots,600,$$

where $\mathbf{c}$ is a vector of constants and $\boldsymbol{\mu}_t$ is a $N(\mathbf{0},\boldsymbol{\Sigma}_{\boldsymbol{\mu}})$ disturbance vector. For parameters $\mathbf{A}$, $\mathbf{c}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\mu}}$, we use estimates based on the data described in Section 4; these estimates are shown in panel A of Table S1. Allowing for a maximum of 15 lags, according to the AIC, BIC and log-likelihood criteria, a first-order VAR is sufficient. Unit root tests indicate that all prices are stationary; this is not surprising since the prices are deflated.

The simulation procedure for trial s is:

1. For $t=1$ set $\log\mathbf{p}_1^{(s)} = \log\mathbf{p}_1$, the vector of observed prices in the first month of our data. Draw $\boldsymbol{\mu}_t^{(s)}$ from $N(\mathbf{0},\boldsymbol{\Sigma}_{\boldsymbol{\mu}})$ and add it to $\mathbf{A}\cdot\log\mathbf{p}_t^{(s)} + \mathbf{c}$ to give $\log\mathbf{p}_{t+1}^{(s)}$.
2. Repeat step 1 for $t=2$ using the generated value $\log\mathbf{p}_t^{(s)}$ to yield $\log\mathbf{p}_{t+1}^{(s)}$.
3. Repeat step 2 for $t=3,\ldots,600$ to give the six simulated series, $\log p_{1t}^{(s)},\ldots,\log p_{6t}^{(s)}$.
4. Apply the BB algorithm to each series to give a set of turning points for episode e, $y_{ie}^{(s)}$, where $e=1,\ldots,E^{(s)}$. Episodes are determined by the turning points of the simulated Divisia price index $DP_t^{(s)} = \Sigma_{i=1}^{6}\bar{w}_{it}^{(s)}Dp_{it}^{(s)}$, where $Dp_{it}^{(s)} = \log p_{it}^{(s)} - \log p_{i,t-1}^{(s)}$ is the log-change in the i^{th} price and $\bar{w}_{it}^{(s)} = 1/2\cdot\left(w_{it}^{(s)} + w_{i,t-1}^{(s)}\right)$ is the value share averaged over months t and t-1. To calculate the simulated value shares, we recover volumes by an inverse demand model of the form $D\log\mathbf{q}_t = \mathbf{B}\cdot D\log\mathbf{q}_t + \boldsymbol{\varepsilon}_t$, where $D\log\mathbf{q}_t = \log\mathbf{q}_t - \log\mathbf{q}_{t-1}$ is the vector of changes in volumes, $D\log\mathbf{p}_t = \log\mathbf{p}_t - \log\mathbf{p}_{t-1}$, and $\boldsymbol{\varepsilon}_t$ is a $N(\mathbf{0},\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}})$ disturbance term. The estimates of $\mathbf{B}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$ are given in panel B of Table S1. The simulated volumes for trial s, $\log\mathbf{q}_t^{(s)} = \left[\log q_{1t}^{(s)},\ldots,\log q_{6t}^{(s)}\right]$ for $t=1,\ldots,600$, are obtained by replacing prices with volumes in steps 1-3 above. Here, we use the estimates of $\mathbf{B}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$; as the off-diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$ are not significant, we simplify the simulation by sampling from a diagonal covariance matrix made up of the diagonal elements of the estimate of $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$. The simulated volumes and prices then define the simulated value shares.
5. The simulated turning points $\mathbf{y}_e^{(s)} = \left[y_{1e}^{(s)},\ldots,y_{6e}^{(s)}\right]$, together with the EM algorithm for missing observations, are then used to estimate model (3.1) by FGLS with a diagonal covariance matrix, as discussed in Section 4. This yields the simulated estimates of the underlying cycle, $\hat{\alpha}_e^{(s)}, e=1,\ldots,E^{(s)}$ episodes, and the phase parameters, $\hat{\boldsymbol{\beta}}^{(s)}$.

The above steps are repeated for s = 1,...,10^4 trials.

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TABLE S1

ESTIMATES OF VAR AND INVERSE DEMAND MODELS

Metal	Aluminium	Copper	Lead	Nickel	Tin	Zinc
A. VAR Model, $\log \mathbf{p}_{t+1} = \mathbf{A} \cdot \log \mathbf{p}_t + \mathbf{c} + \boldsymbol{\mu}_t$						
Coefficient Matrix **A**						
Aluminium	0.860	-0.123	-0.153	-0.159	-0.139	-0.116
Copper	0.024	0.972	0.065	0.081	0.051	0.112
Lead	0.030	0.033	0.917	0.027	0.057	-0.005
Nickel	0.011	0.049	0.072	0.971	0.022	0.007
Tin	-0.056	-0.034	-0.022	-0.100	0.885	-0.082
Zinc	0.035	0.019	0.022	0.050	0.026	0.965
Constant Vector **c**						
	0.805	0.642	0.554	1.171	0.889	0.937
Covariance Matrix $\boldsymbol{\Sigma}_{\boldsymbol{\mu}} \times 100$						
Aluminium	0.318	0.264	0.175	0.258	0.151	0.205
Copper		0.577	0.280	0.377	0.202	0.352
Lead			0.694	0.311	0.180	0.341
Nickel				0.951	0.258	0.321
Tin					0.384	0.144
Zinc						0.590
B. Inverse Demand Model, $\mathrm{D}\log \mathbf{q}_t = \mathbf{B} \cdot \mathrm{D}\log \mathbf{p}_t + \boldsymbol{\varepsilon}_t$						
Coefficient Matrix **B**						
Aluminium	-0.008	-0.122	-0.275	-0.446	-0.241	-0.060
Copper	0.044	0.073	-0.015	0.218	-0.011	0.083
Lead	-0.105	-0.061	0.126	0.088	0.167	-0.109
Nickel	0.001	-0.008	-0.120	-0.113	0.022	-0.031
Tin	0.060	-0.081	0.133	0.080	0.027	-0.016
Zinc	0.147	0.062	0.056	-0.052	-0.027	0.075
Covariance Matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \times 100$						
Aluminium	0.650	0.234	0.204	0.340	0.172	0.296
Copper		0.476	0.220	0.262	0.136	0.180
Lead			0.969	0.295	0.200	0.380
Nickel				1.517	0.313	0.327
Tin					1.025	0.133
Zinc						0.633

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