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A multi-sector model of relatedness, growth and industry clustering

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Abstract

This article builds an understanding of regional innovation specialization by developing a multi-sector model with endogenous growth through quality improving innovations and spillovers from related technologies. The model provides an approach to incorporate the relatedness literature within the mainstream theoretical frameworks of endogenous growth and economic geography. Each firm's technology sector and the location of other firms play a role in each firm's ability to improve its own technology. As a result, firms prefer to co-locate in technologically compatible clusters. Without relying on scale assumptions, the model for the first time coherently links related variety knowledge spillovers to mainstream urban economic frameworks and demonstrates that clustering is possible in both core and peripheral areas.

JEL classifications: R11; O41
Keywords: Innovation; endogenous growth; knowledge spillovers; relatedness; clusters

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1 Introduction

Recent years have seen a rapid rise in research publications based on the principle of relatedness, whereby the pattern of knowledge links between sectors, skills and technologies, and crucially the extent to which they are related to each other, is argued to provide important clues as to the nature and patterns of trade and economic growth at both the national and regional levels (Hidalgo et al., 2018). The relatedness literature has two broad strands of research, namely the product space literature (Hausmann et al., 2007; Hidalgo et al., 2007; Hidalgo & Hausmann, 2009; Figueroa et al., 2018) and the related variety literature (Frenken et al., 2007; Boschma & Iammarino, 2009; Neffke et al., 2011), both of which are implicitly based on a capabilities-type of framework. Although there are some differences between these two approaches in terms of how relatedness and capabilities are measured and analysed, there are also enough similarities and overlaps that for our purposes here we can treat them as simply reflecting different strands of one broad relatedness approach to economic growth and economic geography which has a few key common characteristics. Firstly, economies of scale play no significant role in determining growth patterns, and secondly, neither do standard debates regarding specialization or diversity. Rather it is the technological or network relatedness between different activities, as reflected in the knowledge spillovers or common knowledge underpinnings across different types of activities, sectors and capabilities, that is argued to be crucial for growth.

In the rapidly-growing literature on relatedness two empirical features are apparent. Firstly, regional economic growth is nearly always characterised by relatedness, in that the local sectors experiencing growth are typically connected to each other in terms of their knowledge, technological or skills dimensions. In particular, the relatedness dimensions share some common features such that even though the sectors may differ, to some extent they also overlap in terms of common knowledge characteristics or capabilities. Moreover, these relatedness features also cut across simple explanations of growth based on regional sectoral structure, in that while the distinction between sectoral specialisation and diversity appears to play little role in explaining regional growth (Melo et al., 2009; Beaudry & Schiffauerova, 2009; de Groot et al., 2016), relatedness between local firms and industries appears to be almost always evident in local growth processes (Cicerone et al., 2019). Secondly, in all countries there are successful clusters of economic activity, and especially knowledge-intensive activities, which are located well away from the core highly urbanised areas, and which seem to thrive even without significant scale or proximity effects. At present it is very difficult to account for either of these observations using current urban economic, endogenous growth or new economic geography frameworks.

As it stands, the interest in, and the persuasiveness of, the relatedness approach is based predominantly on many various forms of empirical evidence provided which support theoretical, typically network-based, relatedness arguments (Hausmann & Hidalgo, 2011; Kali et al., 2013; Iacopini et al., 2018; Alshamsi et al., 2018). Yet, at present there are no orthodox theoretical frameworks that link these relatedness arguments to endogenous growth, economic geography or urban economic modelling frameworks. Assuming that the orthodox arguments also reflect many aspects of the growth processes observed in the spatial economy, then the lack of analytical frameworks linking the relatedness arguments to the more orthodox approaches represents a weak link in our current understanding of the spatial economy. Providing such a theoretical framework is the purpose of this article.

As in all endogenous growth models, knowledge spillovers have a vital role. The model described here differs from existing endogenous growth models in that the addition of multiple sectors and locations allows a description of knowledge spillovers between firms that are technologically and spatially separated, but also related to differing degrees. The model we present here for the first time offers a framework to incorporate the concept of technological relatedness with spatial externalities and endogenous growth, and we find that catastrophic agglomeration is not inevitable in spite of factor mobility, because firms in an industry with sufficient own sector knowledge intensity can cluster in a peripheral location. This implies the emergence of sectoral clusters in both agglomerated and peripheral locations, a result which is consistent with the empirical findings of the relatedness literature (Figueroa et al., 2018; Neffke et al., 2011). By developing a model with multiple industries and spillovers based on technological relatedness, firms in peripheral locations balance the forces for clustering in the periphery against forces for agglomeration. Notably it is the sectors with greater own sector knowledge intensity which are more sustainable in a peripheral location than industries with a lower own sector knowledge intensity which are more affected by forces for agglomeration.

The model builds on the theoretical proximity arguments informed by the relatedness literature. Firm innovations, economic development, and regional specialization is described as a branching process (Frenken & Boschma, 2007). In this framework, a firm's ability to develop new innovations is related to both its technological and spatial proximity (Boschma, 2005). Similarly, the density of related varieties also affects a firm's ability to develop new innovations or for a region to diversify (Kali et al., 2013). These features give rise to a network topology of products (Hidalgo et al., 2007; Hidalgo & Hausmann, 2009) based on their technological proximity. This network structure has motivated studies into the dynamics of growth in relation to specialisation and diversification at both national (Hausmann et al., 2007) and regional levels (Boschma & Iammarino, 2009) as well as within the network structure itself (M. A. Fink et al., 2017; Alshamsi et al., 2018). Furthermore, the relatedness literature has now developed a prescriptive policy approach to account for proximity and path dependency in regional and industry policies (Balland et al., 2018; Alshamsi et al., 2018). These types of proximity mechanisms also give technologically related firms an incentive to co-locate in clusters, but the relatedness feature is missing from spatial endogneous growth models despite providing a key mechanism for explaining differences in rates of regional innovation. Complex network relationships are incompatible with orthodox endogenous growth modelling techniques, so much of the empirical research on growth and proximity relies on defining metrics that describe the position of a product or region within the network space (Hausmann & Hidalgo, 2011; Cicerone et al., 2019). Alternatively, this article shows how the technological and spatial proximity features of the relatedness literature can be incorporated into orthodox endogenous growth theory in a relatively straightforward manner.

The rest of the article is organized as follows. Section 2 adds multiple industries and related variety knowledge spillovers to the core-periphery model of growth, Section 3 examines the steady state properties of the model and Section 4 provides a discussion and direction for future research.

2 The model

The model is an extension of standard core-periphery growth models, which are summarised in Bond-Smith & McCann (2014), to now include several industrial sectors with multiple varieties in each and growth without scale effects. This approach enables knowledge spillovers to be described by differing degrees of technological and spatial separation such that firms take account of the knowledge externalities of related varieties. In all other respects, the model follows standard approaches with footloose skilled labour that are briefly described here and fully specified in the appendix. As a result, the catastrophic agglomeration pattern of standard models is nuanced by industry clustering, even in peripheral regions, both in spite of and due to factor mobility.

There are two regions. Goods may be produced and consumed in either region. The two regions are referred to as home and foreign. The model is described for the home region and analogous equations apply to the foreign region. Where it is necessary to specify foreign variables, these are denoted by a tilde ($\tilde{}$) above the variable. There is a traditional goods sector, a manufacturing sector and a competitive research and development sector. The representative consumer has typical intertemporal preferences and standard Euler equations and the transversality condition apply. In specifying the labour market, the model follows Krugman's (1991) modelling trick to equalise wages by setting the worldwide stock of unskilled workers to $1-\mu$ shared equally between regions and the stock of skilled workers to μ. Skilled workers freely migrate between regions in response to wage pressure at the start of each period.

This remainder of this section specifies the specific elements that lead to regional clustering before examining the steady state and discussing the model's implications.

2.1 Multiple industries

In each discrete time period, traditional goods and a variety of manufactured varieties are consumed with a preference for higher quality manufactured varieties.

$$Q_t = C_{T,t}^{1-\mu} \prod_{i=1}^{M} C_{i,t}^{\frac{\mu}{M}}, \quad 0 < \mu < 1. \tag{1}$$

For simplicity, the time subscript t will be suppressed hereafter where the time dimension is clear. Monopolistic competition in each sector is modelled via CES preferences (Dixit & Stiglitz, 1977) for simplicity.

$$C_i = \left[\sum_{j \in n_i, \tilde{n}_i} (A_{i,j} c_{i,j})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{2}$$

where i indicates the sector of variety j (referred to as variety i, j), the factor $A_{i,j}$ represents the quality of variety i, j and $c_{i,j}$ is its quantity consumed. Equations 1 and 2 use discrete variables and assume each n_i is sufficiently large to maintain simplicity, elegance and intuition, but could also be thought of as a continuum of manufacturing varieties, sectors and continuous time (by replacing $\prod_{i=1}^{M}$ with the product integral $\prod_0^M f(x_i)^{di}$ and $\sum$ with the standard integral $\int f(x_j)\, dj$). Similarly, the determinstic model in discrete time can be thought of as equivalent to the expected flow of innovations in a stochastic model in continuous time.

For simplicity, zero transport costs are assumed. This allows the model to focus exclusively on the location and growth effects of technical externalities in research and development. In an extension of the basic model, it is possible to include trade costs or other spatial externalities to demonstrate how firms balance many factors in making location and investment decisions, but here the article focuses only on technical externalities in research and development.

Standard optimization techniques are used to solve for short run equilibrium prices and wages.

2.2 Technology

Production of an individual variety involves a fixed (labour) investment in a quality improving innovation (in the previous period) and a constant marginal cost. Production of each variety is contestable through these quality improvements produced by a competitive research and development sector with free entry. In each period, the quality leader produces variety i, j and potential investors or firms, including incumbents, choose whether to enter in the following period. If a firm enters, it selects a variety and conducts research effort to develop a quality improvement sufficient to gain a niche monopoly position for production in the following period only. Incumbent firms cannot retain the inter-temporal spillover beyond one period of production because free entry in the R&D sector allows each variety to be contestable. Therefore all production firms must invest in a quality improving innovation in the period prior to production and each firm's decision horizon is only one period long. Firms last for as long as they continue to innovate, but it is also possible to consider that each firm dissolves after one period of research and one period of production, to be replaced by a new innovating firm.

Multiple industries and varieties enables a multi-sector knowledge spillover mechanism to be added to the model of endogenous growth without scale assumptions (Young, 1998; Bond-Smith et al., 2018). The fixed cost of manufacturing in the subsequent period t is the skilled labour requirement in the previous period, $t-1$, to achieve the targeted quality level $A_{i,j,t}$ given by:

$$F_{i,j,t-1}(A_{i,j,t}, \bar{A}_{i,j,t-1}) = \begin{cases} \gamma e^{\eta A_{i,j,t}/\overline{A}_{i,j,t-1}} & \text{if } A_{i,j,t} \geq \overline{A}_{i,j,t-1} \\ \gamma e^{\eta} & \text{otherwise,} \end{cases} \tag{3}$$

where γ and η are constants that may be used for calibration and $\overline{A}_{i,j,t-1}$ is an index of technological opportunity for variety i, j, representing the intertemporal spillover of knowledge available to variety i, j researchers. The fixed cost can be thought of as two components: a standard fixed cost of γe^{η} irrespective of quality improvement and a cost of $\gamma e^{\eta A_{i,t}/\bar{A}_{i,j,t-1}} - \gamma e^{\eta}$ for achieving a quality improvement.

The spillover of knowledge between firms is imperfect. $\lambda_R < 1$ is a scalar that describes the proportion of knowledge that is available to a firm that is spatially separated from the location of that knowledge. The same logic of firms being separated by geographic space, can also be applied to manufacturers of different varieties being separated by technological space. Each component of knowledge is also assumed to be weighted according to a related variety approach (Boschma & Frenken, 2009), where the relatedness of technology describes how useful the knowledge is to innovation in a firm's own variety (i.e. proximity in technological space). For simplicity, it is assumed that varieties in the same sector are weighted equally and varieties in other sectors are also weighted equally. Knowledge of a firm's own variety is given a weight of one, knowledge from innovations within the firm's own sector a weight of $\lambda_V < 1$ and knowledge of other sectors a weight of $\lambda_M < 1$ where $\lambda_M < \lambda_V < 1$. If the firm's selected variety was previously produced in the foreign region, it has the same spatial weight as any foreign knowledge $\lambda_R \leq 1$. Evidently, the relatedness of different varieties in the real world is not as simple. To reflect this, it is possible to weight knowledge from every individual pair of varieties by a proximity measure from which firms choose an optimal variety and location (Boschma, 2005), but this additional complexity in a theoretical model of growth is left for future research. For example, Hidalgo et al. (2007); Boschma et al. (2015) and Balland (2016) estimate measures of the proximity of every pair of industries based on a co-occurrance matrix. Maintaining simplicity, these estimates could also be used to calibrate the related and unrelated industry spillover parameters in this model. This technical externality in research and development triggers a "clustering effect" as it induces firms to cluster in locations alongside other firms in their own sector. Firms must also consider this effect alongside incentives to locate in a larger agglomeration where there are more sources of knowledge spillovers from other sectors. This incentive to locate with the larger share of manufacturing, is described as the "agglomeration effect".

The knowledge input to innovation is therefore made up of three components: knowledge of the variety's own quality level, knowledge from within the firm's own sector and knowledge from other varieties in other sectors. It is assumed that knowledge from all sources is additive. For developing a quality improvement to produce in period t, the knowledge spillover that is an input to innovation has three weighted components:

1. the knowledge at time $t-1$ from the firm's own variety i, j, represented the by quality level
$$A_{i,j,t-1} \text{ or } \widetilde{A}_{i,j,t-1}, \tag{3a}$$

2. the weighted average knowledge of quality from varieties within the firm's own sector i weighted by location
$$\overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} = \frac{\sum_{j\in n_i} A_{i,j,t-1} + \lambda_R \sum_{k\in\tilde{n}_i} \widetilde{A}_{i,k,t-1}}{n_i + \tilde{n}_i} \qquad \text{and} \tag{3b}$$

3. a weighted average knowledge of quality improvement from other manufacturing sectors weighted by location
$$\overline{A}_{i\forall M,t-1} = \frac{\sum_{m\in M,\, j\in n_m} A_{m,j,t-1} + \lambda_R \sum_{m\in M,\, k\in\tilde{n}_m} \widetilde{A}_{m,k,t-1}}{\sum_{m=1}^{M} n_m + \sum_{m=1}^{M} \tilde{n}_m}, \tag{3c}$$

where A describes the quality improvement in each period and λ_R represents the weighting for knowledge that is sourced from firms in a different location than the firm producing variety i, j. Note that in the steady state with zero transport costs, the firms in each sector are clustered in either the home region or the foreign region, but not both. Therefore, each of these components will include only the home region variables or the foreign region variables. In an unsteady state, or between steady states, both types of variables could be included.

For a home region firm producing variety i, j, the overall index of technological opportunity is given by:

$$\overline{A}_{i,j,t-1} = \max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) + \lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}. \tag{3d}$$

That is, the index of technological opportunity is the knowledge associated with the latest innovations in the firm's own variety weighted by location plus a weighted average of the knowledge associated with innovations of all other varieties weighted by location and technological relatedness. As a result, firms may face a trade-off in the costs of innovation between locating in a cluster of technologically related firms or locating in an agglomeration of relatively unrelated firms. It is this trade-off which leads to the possibility of including clusters in an endogenous growth model and offers amenable implications for regional innovation and growth policy.

It is assumed that the number of sectors is fixed such that there are always M sectors. The number of varieties in each sector is determined by the parameters of the model and the distribution of skilled workers and industries. It is possible for new varieties to replace existing varieties or to expand the range of varieties in an industry. If the variety has never been produced before, the knowledge of a firm's own innovations is replaced by a weighted average of innovations for its selected sector i. This maintains symmetry in each sector even when a new variety is introduced. The index of technological opportunity for new varieties is given by:

$$\begin{aligned}\overline{A}_{i,j,t-1} &= \frac{\sum_{j\in n_i} A_{i,j,t-1}+\lambda_R \sum_{k\in\tilde{n}_i} \widetilde{A}_{i,k,t-1}}{n_i+\tilde{n}_i} + \lambda_V \overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1} \\ &= (1+\lambda_V)\,\overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}.\end{aligned} \tag{3e}$$

This specification means that each firm in the same location faces the same costs of improving an existing variety in that region or introducing a new variety. As a result, no two firms choose the same variety because monopoly profits are always greater than individual duopoly profits. Analogous equations exist for foreign firms.

2.3 Innovation

Based on standard techniques for constrained optimisation firms select a quality improvement where the elasticity of research cost with respect to quality is equal to the elasticity of demand with respect to quality.

$$\varepsilon^{c_{i,j,t}}_{A_{i,j,t}} = \varepsilon^{F_{i,j,t}}_{A_{i,j,t}} \tag{4}$$

Rearranging Equation 4 obtains:

$$\frac{\sigma-1}{\eta} = \frac{A_{i,j,t}}{\overline{A}_{i,j,t-1}}, \tag{5}$$

which describes the preference of firms to invest in quality improvement. By substitution into Equation 3, the cost of innovation or preference to invest and the number of skilled workers employed in research by each firm per period is:

$$F_{i,j,t} = \gamma e^{\frac{\eta A_{i,j,t}}{\overline{A}_{i,j,t-1}}} = \gamma e^{\sigma-1}. \tag{5a}$$

Firms select a quality target of:

$$A_{i,j,t} = \frac{\sigma-1}{\eta}\left[\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) + \lambda_V \overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}\right]. \tag{5b}$$

This is a quality improvement multiplier of:

$$\frac{A_{i,j,t}}{\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right)} = \frac{\sigma-1}{\eta}\left[1+\frac{\lambda_V \overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}}{\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right)}\right]. \tag{5c}$$

Assuming this multiplier is always greater than one, there are always quality improvements in equilibrium.

Quality improvement per period is given by:

$$I_{i,j,t} = A_{i,j,t} - \max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) =$$
$$\left(\frac{\sigma-1}{\eta}-1\right)\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) + \frac{\sigma-1}{\eta}\left[\lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}\right]. \tag{6}$$

Intuitively Equation 6 (also Equations 5b and 5c) has two components. Quality improvement is made up of the innovation from direct investment in R&D (or R&D-based innovation):

$$\left(\frac{\sigma-1}{\eta}-1\right)\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) \tag{6a}$$

plus the quality improvement due to the variety specific knowledge spillover:

$$\frac{\sigma-1}{\eta}\left[\lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}\right] \tag{6b}$$

giving the total quality improvement as given in Equation 6.

2.4 Labour market clearing

Labour market clearing requires that the total labour used in home region manufacturing (L_M) and R&D (L_R) are equal to the total supply of regional skilled workers (L_K). In equilibrium, the skilled labour used in manufacturing in the home region is the worldwide expenditure on manufactured goods produced in the home region divided by the price per unit and multiplied by its marginal cost:

$$L_M = \frac{\mu S\left(E+\tilde{E}\right)}{p}\beta = \frac{\sigma-1}{\sigma}\mu S\left(E+\tilde{E}\right), \tag{7}$$

where $S = \frac{\sum_{i\epsilon M} n_i p_i c_{i,}}{\sum_{i=1}^{M} n_i p_i c_{i,} + \sum_{i=1}^{M} n_i \tilde{p}_i \tilde{c}_i} = \frac{\sum_{i\epsilon M} n_i p_i c_{i,}}{\mu\left(E+\tilde{E}\right)}$ is the total market share of manufacturing expenditure held by home region firms. The labour used in research is equal to the number of firms in the next period multiplied by the investment in research labour by each individual firm:

$$L_{R,t} = \gamma e^{\sigma-1}\sum_{i=1}^{M} n_{i,t+1}. \tag{8}$$

Home region skilled labour market clearing in period t therefore requires:

$$L_{K,t} = \frac{\sigma-1}{\sigma}\mu S\left(E+\tilde{E}\right) + \gamma e^{\sigma-1}\sum_{i=1}^{M} n_{i,t+1}. \tag{9}$$

Analogous equations exist for foreign region manufacturers. Labour market clearing, the firm profit function and the free entry relation can be applied using standard techniques to solve for the equilibrium and steady state number of varieties in each industry and region.[1]

3 Steady state

Firms exist where there are skilled workers to be employed and it is assumed there is no unemployment. Furthermore, with zero transport costs, all wage pressure in the model is a result of the costs and benefits of firm location decisions. Workers migrate in response to wage pressure until a steady state is reached where wage pressure has dissipated or all skilled workers and their employing firms agglomerate in a single region with the highest wage. In the steady state, wage pressure has dissipated such that all skilled workers and firms prefer their present location and could not find a higher wage by switching location individually. Analysis of the steady state proceeds on the basis of comparing alternative location choices for firms, which influences wage pressure and migration, leading to the steady state. To minimise their labour cost for innovation, firms form alongside other firms in the same sector to maximise the knowledge available for innovation. This is the mechanism for innovation clustering. In the steady state no firm or its workers want to leave their present location because access to knowledge spillovers is maximised by remaining in that location, holding the location of all other firms constant. In unsteady states, greater knowledge spillovers are available to some firms in a sector, leading to wage pressure. Steady states exist with clusters of firms from the same sector in a peripheral region (i.e. a region with a smaller share of manufacturing) if the sector has a sufficient own sector knowledge intensity that firms prefer the peripheral location over relocating to the agglomerated region in order to access knowledge spillovers from the cluster.

3.1 Requirements for switching firm

For a home region firm located alongside all other firms in the same sector considering a switch to the foreign location, the function for the preferred investment in innovation in the new location is the same as in Equation 5a (with notation $\tilde{F_H}$), but with the knowledge input to innovation adjusted by the new location of the firm:

$$\tilde{\overline{A}}_{i,j,t-1} = \lambda_R A_{i,j,t-1} + \lambda_V \widetilde{\overline{A}}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \widetilde{\overline{A}}_{i\forall M,t-1}. \tag{10}$$

When located in the foreign region the firm ideally also prefers to select a quality improvement where the elasticity of research cost with respect to quality is equal to the elasticity of demand with respect to quality. Elasticities are the same in either region, so Equation 5 is the same for foreign firms. In assessing the costs and benefits of each alternative location, contestability and the free entry criteria requires the firm achieves the greatest quality improvement available from the alternative location choices in order to participate in the market in period t. As a result, firms in the location that receives the highest knowledge spillover for that industry determine the quality target required for entry. That is, $A_{i,j,t} = \max\left(\frac{\sigma-1}{\eta}\overline{A}_{i,j,t-1}, \frac{\sigma-1}{\eta}\tilde{\overline{A}}_{i,j,t-1}\right)$. In unstable states, firms in a location with lower knowledge spillovers require additional skilled workers to achieve the target (i.e. $F_{i,j,t} > \gamma e^{\sigma-1}$) and must offer a lower market clearing wage than the other region in order to satisfy the free entry condition. Anything less than this will mean a new firm can create a greater innovation in the higher technology region and take the market from the incumbent.

In the real world, firms would also consider factor prices, trade costs and the value of sales in each location in addition to those factors considered here. Greater value of sales or lower factor prices could justify a firm choosing a location that is suboptimal for R&D (or a firm choosing a more optimal location for R&D despite a suboptimal location for factor prices or sales), but it is the balance of these which determines the overal optimal location. These additional factors complicate the model and are therefore left aside to focus only on knowledge externalities in research.

3.2 The requirements for a steady state

This section considers possible distributions of manufacturing and research (both steady and unsteady states) where migration of skilled workers due to the spatial inequality of wages and the switching location of firms due to differences in knowledge spillovers lead to the steady state. The steady state is defined as constant regional division of economic activity and population. In such a steady state there is constant investment in R&D and a constant quality improvement from R&D based innovation, as defined by Equation 6a, but there may be declining or increasing diffusion-based innovation (Equation 6b). This is a steady state, because firms and workers have no incentive to switch region between periods and therefore the distribution of economic activity is "steady". This definition of a steady state is required, because sectors with a relatively higher quality level A achieve lower rates of quality improvement from diffusion than sectors with a low quality level A and therefore, the spillovers from other sectors change over time. This is a similar relationship to that discussed in the distance to frontier literature (Acemoglu et al., 2006), but focused on the relatedness between different varieties.

Consider an unsteady state with a cluster of firms in the home region defined by a relatively greater number of firms in the same industrial sector i locating in the home region ($n_i > \tilde{n}_i$). In an unsteady state, Section 3.1 implies a greater research effort for foreign region firms in sector i to enter the market ($\tilde{F}_{i,j,t} > \gamma e^{\sigma-1}$) such that the number of researchers per firm varies between regions, but in regions where a firm requires more skilled workers to achieve the innovation target for entry, firms offer a lower market clearing wage to avoid losses. A home region firm in that cluster will only switch if the firm can achieve greater return on investment in the new location:

$$\frac{\tilde{V}}{\tilde{w}\tilde{F}} > \frac{V}{wF}, \tag{11}$$

where V is defined as price less marginal cost multiplied by total production in one period. Therefore, a firm will choose the location where the cost of innovation is the lowest, driving wage pressure and the migration of workers, because a lower wage would be offered if there is lower knowledge spillovers. With migration driven by wage pressure, unequal wage rates are not sustained because migration equalises wages and reinforces the location of switching firms. Wages, prices and value equalise between locations in the steady state. The requirement for a firm to remain located in the home region simplifies to:

$$\tilde{F} \geq F, \qquad \forall j, i, \tag{11a}$$

although location changes to reach the steady state could take multiple periods because migration does not happen instantaneously.

All firms in the same sector and location have the same cost of innovation. Each individual firm is small relative to the size of the entire market, so it is assumed that individual firms do not account for any effect on wages from switching location. If Equation 11a does not hold true for one firm in sector i such that the firm switches location, it will also not hold true for other firms in sector i (even more so after the first firm switches) such that all home region firms in the sector will also eventually switch location. In addition, if a sector were shared equally across two regions, a single firm switching means one region would now have the larger share of industry and Equation 11a would no longer hold true for firms remaining in the original location. As a result of these ad hoc dynamics, each sector will remain clustered in one location in the steady state, determined by hysteresis, until Equation 11a no longer holds.

Knowledge spillovers are greater with industry clustering (i.e. concentration of firms in the same sector $n_i > \tilde{n}_i$), and with agglomeration (i.e. concentration of sectors $\sum_{i \epsilon M} n_i > \sum_{i \epsilon M} \tilde{n}_i$). These two factors determine firm loation. Firstly, firms prefer locations with a greater share of their own sector such that firms in each sector cluster in a single location - the so-called "clustering effect." But firms must balance this attraction with a preference to locate where there are more firms overall, because greater concentration of all manufacturers also increases knowledge spillovers. This alternative force for firm concentration with all manufacturing firms is described as the "agglomeration effect". Depending on the distribution of each sector, these forces may be in the same direction or could be in opposite directions. Sectors that cluster in the smaller region may still sustainably produce in that location if the clustering effect is greater than the agglomeration effect, because the clustering force is in the opposite direction. This scenario is described as a "peripheral cluster".

As described above, the quality improvement required for entry is set by the highest level of quality from either region that is available for the fixed cost of $\gamma e^{\sigma-1}$. Assume this quality level is obtained in the home region such that $\tilde{\overline{A}}_{i,j,t-1} < \overline{A}_{i,j,t}$ and $F = \gamma e^{\sigma-1}$. The cost of achieving the quality level $A_{i,j,t}$ for a firm that is switching to the foreign location is:

$$\tilde{F} = \gamma e^{\frac{\eta A_{i,j,t}}{\tilde{\overline{A}}_{i,j,t-1}}} \tag{12}$$

Firms select a quality target determined in the home location, given by Equation 5b. Analogous equations exist if the foreign region is the technology leading region for variety j. The intertemporal spillover of the firm's own knowledge diminishes by $1-\lambda_R$ when the firm switches. Substituting the knowledge input, modified for the foreign region (10), and the targeted quality level (Equation 5b) into Equation 12 gives:

$$\tilde{F} = \gamma e^{(\sigma-1)\frac{A_{i,j,t-1}+\lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1}+\lambda_M \overline{A}_{i\forall M,t-1}}{\lambda_R A_{i,j,t-1}+\lambda_V \tilde{\overline{A}}_{i\forall(n_i+\tilde{n}_i),t-1}+\lambda_M \tilde{\overline{A}}_{i\forall M,t-1}}}. \tag{12a}$$

The difference between entry costs in the home region F and entry costs in the foreign region $\tilde{F}$ is the exponent in $\tilde{F}$ is multiplied by the ratio of knowledge spillovers in each location alternative, where the weightings depend on the current locations of other firms. If foreign knowledge spillovers are lower, there will be a greater cost of innovation in the foreign region as given by Equation 12a. Substituting (12a) and $F = \gamma e^{\sigma-1}$ into Equation 11a and rearranging shows that in the steady state, the firm chooses the location where knowledge spillovers are greater. In this case firms choose the home location, because

$$A_{i,j,t-1} + \lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1} \geq \lambda_R A_{i,j,t-1} + \lambda_V \tilde{\overline{A}}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \tilde{\overline{A}}_{i\forall M,t-1}. \tag{13}$$

It can be seen that the inequality holds for two types of sectoral steady states. In the first type, all varieties and industries are agglomerated in a single location, determined by hysteresis. The clustering effect from locating alongside producers of technologically related varieties (i.e. the same sector) and the agglomeration effect from locating alongside other manufacturers, are both in the same direction towards a single agglomerated location. An alternative scenario where varieties in each industrial sector are split equally between the two locations is not a steady state, but a knife-edge, because if a single firm were to switch locations due to ad hoc dynamics, that location that would have marginally higher knowledge spillovers and therefore all firms would also eventually switch to the larger region. The second type of steady state is where each industrial sector is clustered in a single location and sectors are shared between locations. In this type of steady state sectors may not be shared equally between regions, because the clustering effect for firms in industry clusters in a peripheral region may be greater than the agglomeration effect.

3.3 The requirements for steady state peripheral clusters

With symmetry and clustering of all varieties in each sector in the steady state, the inequality becomes much simpler such that $A_{i,j,t-1}$ for all varieties in sector i and $\overline{A}_{i\forall(n_i+\tilde{n}_i),t-1}$ can be denoted as a quality parameter for any randomly selected variety in sector i, $A_{i,t-1}$. By the nature of Cobb-Douglas preferences specified in Equation 1, each sector in aggregate contributes equally to utility. Relative quality magnitudes between sectors describe the knowledge intensity of each sector as an input to innovation relative to the knowledge inputs from other sectors. This comparative technology measure is described as "own sector knowledge intensity" and it is expressed by a relatively higher A_i for sector i. That is, if sector i has a higher own sector knowledge intensity, it means firms in sector i source a higher share of knowledge from within their own sector compared to firms in other sectors who source a lower share from their own sectors. By substituting 3c the inequality can be rearranged to describe a knowledge intensity threshold for sector i to produce sustainably in the home region:

$$A_{i,t-1} \geq \frac{\lambda_M}{1+\lambda_V} \frac{\sum_{m\epsilon M} \widetilde{n}_m \widetilde{A}_{m,t-1} - \sum_{m\epsilon M} n_m A_{m,t-1}}{\sum_{m\epsilon M} n_m + \sum_{m\epsilon M} \widetilde{n}_m}. \tag{13a}$$

If sector i has a quality target greater than this threshold level, it is possible for sector i to be clustered in the home region in the current period, even if the home region is not the location of other sectors. If all other sectors are agglomerated in the foreign region, this increases the threshold for the quality parameter in sector i. If this threshold is satisfied for all sectors, this is a steady state, because no single firm will switch location in the coming period, wages are equal across locations so there is no change in labour endowments in each region, all firms will grow at the same rate in each industry and will continue to grow in future periods.

In the steady state, this threshold property is easy to test for each variety, because, to be met in all sectors, it only needs to be tested for the variety (or sector) with the lowest technology level in each location. If technology in any single sector is below this threshold, this sector will switch region and the relevant threshold will be redetermined.

3.4 The steady state in the long run

The steady state was defined such that a distribution of economic activity is sustainable indefinitely. Therefore, this technology parameter threshold must be met indefinitely for the distribution to be a steady state. The last case to consider is whether greater innovations in the agglomerated sectors in the foreign region lead the threshold to grow faster than quality in the peripheral cluster. That is, equation 13a must be met for all time periods. If it is not met in the current period (for the innovations that occurred in $t-1$), the firm will switch. There are two clear steady states. The core-periphery outcome is where all sectors cluster in a single region and is a long-run steady state where all firms benefit from co-locating. Alternatively, the equal distribution outcome where half the sectors are clustered in each region is a steady state if there is also an equal distribution of technology intensities. Each location will have equal growth in the quality levels of comparable technology-intensive industries, so there will be no incentive for firms or workers to switch location during any time period.

A third type of steady state, the peripheral cluster equilibrium, where clusters of firms in the same sector(s) are located in the region with a smaller share of all industrial sectors, is also possible to be a steady state if the increases in quality levels in the peripheral cluster are greater than or equal to the change in the threshold which enables the peripheral cluster to continue in the coming period. This allows the threshold to hold in subsequent periods. Consider how the technology threshold changes over time. Taking the discrete derivative of the threshold (Equation 13a), there is an additional threshold that determines whether the distribution is a long-run steady state:

$$\Delta_t A_{i,t-1} \geq \Delta_t \left(\frac{\lambda_M}{1+\lambda_V} \frac{\sum_{m\epsilon M} \widetilde{n}_m \widetilde{A}_{m,t-1} - \sum_{m\epsilon M} n_m A_{m,t-1}}{\sum_{m\epsilon M} n_m + \sum_{m\epsilon M} \widetilde{n}_m} \right). \tag{13b}$$

The discrete derivative of the quality target function (5b) with respect to time yields:

$$\Delta_t A_{i,t-1} = I_{i,t} = \left(\frac{\sigma-1}{\eta} - 1 \right) A_{i,t-1} + \lambda_V \overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}. \tag{13c}$$

Substituting this into the differentiated inequality (Equation 13b) and rearranging gives:

$$I_{i,t} \geq \frac{\lambda_M}{1+\lambda_V} \left(\frac{\sum_{m\epsilon M} \widetilde{n}_m \widetilde{I}_{m,t} - \sum_{m\epsilon M} n_m I_{m,t}}{\sum_{m\epsilon M} n_m + \sum_{m\epsilon M} \widetilde{n}_m} \right). \tag{13d}$$

As long as the size of innovation is greater than the difference between the aggregate innovations in either region, divided by the total number of firms and multiplied by $\frac{\lambda_M}{1+\lambda_V}$, sector i can last indefinitely in a peripheral cluster.

Since $I_{i,t} = A_{i,t} - A_{i,t-1}$, the thresholds can be combined (Equations 13a and 13d):

$$\begin{aligned} \frac{\lambda_M}{1+\lambda_V} \frac{\sum_{m\epsilon M} \widetilde{n}_m \widetilde{A}_{m,t-1} - \sum_{m\epsilon M} n_m A_{m,t-1}}{\sum_{m\epsilon M} n_m + \sum_{m\epsilon M} \widetilde{n}_m} \leq A_{i,t-1} \leq \\ A_{i,t} - \frac{\lambda_M}{1+\lambda_V} \left(\frac{\sum_{m\epsilon M} \widetilde{n}_m \left(\widetilde{A}_{m,t} - \widetilde{A}_{m,t-1}\right) - \sum_{m\epsilon M} n_m (A_{m,t} - A_{m,t-1})}{\sum_{m\epsilon M} n_m + \sum_{m\epsilon M} \widetilde{n}_m} \right). \end{aligned} \tag{13e}$$

Since Equation 13a is already satisfied, Equation 13e can be rearranged to:

$$A_{i,t} \geq \frac{\lambda_M}{1+\lambda_V} \frac{\sum_{m\epsilon M} \widetilde{n}_m \widetilde{A}_{m,t} - \sum_{m\epsilon M} n_m A_{m,t}}{\sum_{m\epsilon M} n_m + \sum_{m\epsilon M} \widetilde{n}_m}. \tag{13f}$$

This is the same as the earlier threshold advanced one period. Therefore, if the threshold is met for technology levels in the current period for the marginal industries (one in each region), it will also be met for all industries in all future periods. As a result, whenever the threshold is met for all sectors, the distribution of technology and economic activity is a steady state.

3.4.1 Summary of steady states

Three possible steady states have been derived:

1. Equal Distribution: even distribution of technology and number of sectors per region.

2. Core-Periphery: all industry agglomerates in a single region.
3. Peripheral Cluster: an industry that is own industry technology intensive produces sustainably in the periphery. A home region peripheral cluster in sector i must have a knowledge input to innovation that satisfies:

$$A_{i,t-1} \geq \frac{\lambda_M}{1+\lambda_V} \frac{\sum_{m \epsilon M} \widetilde{n}_m \widetilde{A}_{m,t-1} - \sum_{m \epsilon M} n_m A_{m,t-1}}{\sum_{m \epsilon M} n_m + \sum_{m \epsilon M} \widetilde{n}_m}.$$

4 Discussion: The impact of knowledge spillovers

Consider how varying the knowledge spillover parameters λ_R, λ_V and λ_M affects technology improvement and the distribution of economic activity. Increasing λ_R increases the level of knowledge transfer between locations. Economic integration which increases the ability to transfer knowledge between locations is growth-enhancing. If there is a peripheral cluster (either medium or long-term) or equal distribution steady state, firms benefit from the additional transfer of knowledge between locations which boosts all firms' abilities to improve technology. The impact is greater for a region with a smaller share of manufacturing, because a greater share of their technology improvement comes from inter-regional knowledge spillovers than for the agglomerated region. This result is consistent with results found by Baldwin et al. (2003) where knowledge spillovers are growth-enhancing.

Baldwin & Forslid (2000) found that increasing regional knowledge spillovers is stabilising for equal distribution outcomes, because it allows the equal distribution to remain a steady state for a larger range (at the lower end) of transport costs. Similarly it was found that regional knowledge spillovers are destabilising for the core-periphery outcome. Bond-Smith et al. (2018) had a similar conclusion regarding stability with the addition that the consequences in the quality ladders model may be more catastrophic than in the product variety model because varieties switch location. Since transport costs are assumed zero in this model, stability is considered in terms of the effect on the steady state threshold of peripheral clusters, from changing each of the knowledge spillover parameters, λ. Changing λ_R has no effect on the steady state threshold as described in Equation 13a, so with the definition of stability used here, λ_R is neither stability enhancing or diminishing. However the threshold is affected by λ_V and λ_M.

λ_M increases the steady state threshold (Equation 13a). This implies that increases in λ_M are destabilising, because they could trigger a change in the steady state. Policies which increase the ability for knowledge to transfer between sectors has two effects, it makes both locations more attractive by increasing the knowledge available for technology improvement, but it has a greater effect on the region with a greater share of industry. Consequently, increases in λ_M reduce the relative own sector knowledge intensity of each sector, making lower knowledge intensive sectors more likely to switch to the agglomerated location.

λ_V decreases the threshold for a similar related reason. Increasing λ_V makes locating alongside other firms in their own sector more valuable. As a result, it increases the relative own sector knowledge intensity of each sector, making lower knowledge intensive sectors less likely to switch to the agglomerated location. Therefore, λ_V is stability-enhancing due to increasing the benefits from the clustering of related technology firms. This result implies that peripheral regions which specialise in particular industries and diversify into related industries based on common capabilities are more resilient to economic shocks that could otherwise trigger catastrophic agglomeration processes.

These results capture many of the features of empirical research on regional growth and regional policy frameworks. For example regional economies exhibit patterns of relatedness (Boschma & Frenken, 2009) and the growth path of regions depends upon relatedness (Neffke et al., 2011). This would be particularly clear in an expanded framework in which λ_M differs by varying degrees for every pair of industries, but the degrees of relatedness here are sufficient to intuitively see these regional growth characteristics. In this way, the model captures the implications of the product space(Hidalgo et al., 2007) for regional economies (Cicerone et al., 2019). The model also captures many of the principles of regional development policies (McCann & Ortega-Argilés, 2015).

5 Directions for future research

The model presented here offers a framework to consider related technology spillovers and the role of clustering for firm location and innovation decisions that can also be applied to other modelling techniques. The knowledge spillover and relatedness properties are parsimoniously captured by just three parameters λ_R, λ_V, and λ_M. There is now a growing body of empirical research that can be used to calibrate the parameters λ_R, λ_V, and λ_M. There is a growing literature on connectivity (McCann & Acs, 2011) and diffusion (Haldane, 2018) that can be used to calibrate λ_R. In terms of calibrating λ_V and λ_M, there are increasing numbers of papers that give regional weightings. In most of the entropy-based related variety literature the weightings of the knowledge flows are determined by the structural employment shares of each of the sectoral categories, and from the empirical research of scholars such as Koen Frenken, Ron Boschma, Frank Van Oort, Frank Neffke and their co-authors, there are now estimates for many different countries and regions. Meanwhile, following (Teece et al., 1994), other types of frameworks use coherence measures of co-occurrence as weightings for knowledge flows (Rocchetta & Mina, 2019). Furthermore, regional applications of the Hausmann-Hidalgo types of complexity measures (Daboín et al., 2019; Escobari et al., 2019; Mealy et al., 2019; Mealy & Coyle, 2019) use network-centrality models based on the conditional probabilities of specialisation to weight these knowledge spillover values, although these latter models need to be adjusted using bootstrapping techniques to make them meaningful at the regional level (Cicerone et al., 2019). Delgado et al. (2016) use similar relatedness metrics to define types of clusters that alternatively can be used to assign firms as related-industry members of each cluster. As such the model offers a technique to integrate the empirical findings in the relatedness literature within the theoretical frameworks of endogenous growth and new economic geography. There are a number of implications that are unique to this approach offering insights for regional growth beyond the existing orthodox frameworks.

If there were also a stochastic aspect to the model, such as a probability of R&D also developing a new variety or quality improvement in an alternative sector, in addition to the expected quality improvement in the firm's own variety, this could lead to the emergence of new peripheral clusters and the constant shifting of new peripheral clusters between peripheral locations and agglomerated locations. This is an example of using the modelling techniques in Duranton (2007) or Brezis & Krugman (1997) as an additional extension to the model presented here.

The stochastic emergence of alternative or replacement varieties, even in peripheral locations, can be thought of as the historical events that emerge prior to the model described here as well as an ongoing churn of industry as in the original models by Duranton (2007) and Brezis & Krugman (1997). Therefore the results of such a hybrid model can be implied by the results of all three models. As with Duranton (2007) or Brezis & Krugman (1997) it could be expected that there will be switching of industry between locations but by adding a technological relatedness approach this is now partially endogenous switching from peripheral to core locations and partially stochastic churning of industry between locations. The framework here is consistent with these models, but provides an additional richness of endogenously sorting industries between peripheral and core locations due to technical externalities and sectoral knowledge intensity. Both of these stochastic models (Brezis & Krugman, 1997; Duranton, 2007) explain the rise and fall of locations through the stochastic emergence of new technologies in new locations, but fail to explain why a peripheral location might not be an optimal choice for some industries.

Combining Duranton (2007) or Brezis & Krugman (1997) with the model presented here, is expected to suggest that new industries are most likely to emerge in already agglomerated locations, but peripheral clusters will remain part of the economic landscape, developing new peripheral clusters, but at a lower frequency than core locations. Of the industries that emerge in the periphery, only the sectors with a level of own sector technology intensity greater than the relevant threshold can remain sustainable in the medium or long term. Furthermore, the emergence of innovations in an alternative sector is likely to be in related industries, so such a framework would help to explain both regional branching (Boschma & Frenken, 2009) and the sorting of sectoral clusters between peripheral and agglomerated locations as well as support regional policy frameworks based on relatedness (Balland et al., 2018).

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Appendix: The model

Consider an economy with two production sectors: traditional and manufactured goods. Each has its own factor of production: unskilled and skilled labour respectively. There is also an innovation sector that develops quality improvements in manufactured varieties using skilled labour and knowledge spillovers that are subject to spatial and varietal characteristics. Technological knowledge related to all manufactured varieties is an input to research and development, but knowledge spillovers do not transfer perfectly between locations or varieties. The usability of knowledge for innovation is associated with the spatial and technological proximity of related varieties: there is a greater ability for firms to use knowledge of varieties produced in the same location or within the same technological sector. Firms which choose to locate alongside other manufacturers with technologically related varieties therefore have greater access to technological knowledge and a lower cost of quality improvement. This factor is the mechanism responsible for industrial clustering.

Model specification

There is a traditional goods sector, a manufacturing sector and a competitive research and development sector. The manufacturing and research sectors both employ mobile skilled labour. Unskilled labour is immobile and is employed in the traditional goods sector which is included in the model to ensure trade, even with full manufacturing agglomeration. Unlike similar models, manufacturing is extended to many sectors. The utility function uses a Cobb-Douglas function to describe preferences between traditional goods and manufacturing sectors with Dixit-Stiglitz preferences in each sector. To keep the model tractable and without loss of generality, there are assumed to be two locations where firms and workers (consumers) can choose to locate. In order to focus on the technical externalities associated with innovation clustering, transport costs are assumed zero.

Preferences

The representative consumer has a taste for traditional goods and a range of manufactured varieties from several industrial sectors with a preference for higher quality manufactured varieties. The two regions are referred to as home and foreign. The model is described for the home region and analogous equations apply to the foreign region. Where it is necessary to specify foreign variables, these are denoted by a tilde ($\tilde{}$) above the variable. With $\rho > 0$ as the discount rate, the representative consumer has inter-temporal preferences given by:

$$U = \sum_{t=0}^{\infty} \alpha^t \ln Q_t, \quad \alpha = 1/(1+\rho), \tag{1}$$

where Q_t is consumption of traditional goods produced in either region (C_T) and manufactured goods from M sectors in period t with Cobb-Douglas preferences:

$$Q_t = C_{T,t}^{1-\mu} \prod_{i=1}^{M} C_{i,t}^{\frac{\mu}{M}}, \quad 0 < \mu < 1. \tag{1a}$$

For simplicity, the time subscript t will be suppressed hereafter where the time dimension is clear. Monopolistic competition in each sector is modelled via CES preferences (Dixit & Stiglitz, 1977) for simplicity.

$$C_i = \left[\sum_{j \in n_i, \tilde{n}_i} (A_{i,j} c_{i,j})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{1b}$$

where subscript i indicates each sector, subscript j refers to each of the individual varieties j in sector i, the factor $A_{i,j}$ represents the quality of variety j in sector i and $c_{i,j}$ is its quantity consumed. Ottaviano et al. (2002) point out the limitations of CES preferences in a trade model, but the focus here is the inputs to innovation and the outcomes for growth rather than retail price competition. Other forms of competition may yield a more realistic picture of price competition, but would have similar implications for innovation and firm location and would involve a more complicated derivation. Varieties can be purchased from domestic manufacturers or imported from foreign manufacturers, where n_i and $\tilde{n}_i$ are the number of home and foreign manufacturers in sector i. Symmetry in prices, output and quality levels is assumed for all varieties in each sector i, meaning the variety subscript j can be suppressed and Equation 1b can be simplified to:

$$C_i = \left[n_i (A_i c_i)^{\frac{\sigma-1}{\sigma}} + \tilde{n}_i \left(\tilde{A}_i \tilde{c}_i \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{1c}$$

where A_i and c_i refer to the quality and quantity of any single symmetric variety in sector i. While each sector is assumed symmetric, symmetry is not assumed across sectors and some sectors may have higher or lower measures of quality (and knowledge).

Equations 1a and 1b use discrete variables and assume each n_i is sufficiently large to maintain simplicity, elegance and intuition, but could also be thought of as a continuum of manufacturing varieties, sectors and continuous time (by replacing $\prod_{i=1}^{M}$ with the product integral $\prod_0^{M\, di}$ and $\sum$ with integral signs $\int$). The determinstic model in discrete time can be thought of as equivalent to the expected flow of innovations in a stochastic model in continuous time.

Intertemporal utility optimisation implies the transversality condition and Euler equation:

$$\frac{E_t}{E_{t-1}} = \frac{1+r}{1+\rho}, \tag{2}$$

where E_t is consumer expenditure in period t, ρ is the rate of time preference and r is the rate of return on savings between periods $t-1$ and t. The equation applies to all workers in both locations. World expenditure is normalised in each period t such that, $E + \tilde{E} = 1, \forall t$.

For simplicity, zero transport costs are assumed. This allows the model to focus exclusively on the location and growth effects of technical externalities in research and development. In an extension of the basic model, it is possible to include trade costs or other spatial externalities to demonstrate how firms balance many factors in making location and investment decisions, but here the paper focuses only on technical externalities in research and development.

Labour

In specifying the labour market, the model follows Krugman (1991) by setting the worldwide stock of skilled workers to μ and the stock of unskilled workers to $(1 - \mu)$, shared equally between regions. With L_T and $\tilde{L}_T$ the supply of unskilled workers in the home and foreign regions respectively is:

$$L_T = \frac{1-\mu}{2}, \qquad \tilde{L}_T = \frac{1-\mu}{2}, \tag{3}$$

so that total production of traditional goods is shared equally between regions. Unskilled workers cannot migrate between locations. The choice of units (μ skilled workers and $1 - \mu$ unskilled workers) is a normalisation that ensures prices and wages in the traditional goods sector are the numéraire and that the wage rate of skilled workers equals that of unskilled workers. If the number of skilled workers were specified differently, the wages of skilled workers are a constant multiple of the wage rate of unskilled workers. Similarly, a portion of skilled labour could be specified as highly skilled to work in research and development only (as in Forslid & Ottaviano (2003)) and the wages of highly skilled workers would be a constant multiple of the skilled and unskilled workers' wages. Heterogenous workers, education and skill levels are not the focus of the model. Simplicity is maintained by avoiding these additional multiples using Krugman's (1991) modelling trick. A scaling factor could also be used to calibrate the model to any arbitrary growth or wage rate.

L_K describes the number of skilled workers in the home region (subscript K describes the "knowledge sectors" of manufacturing and research and development) such that:

$$L_K + \tilde{L}_K = \mu. \tag{4}$$

Manufacturing and research and development are subject to footloose labour where skilled workers migrate freely in response to wage pressure $w_{K,t} - \tilde{w}_{K,t}$ at the start of each period. If there are wage differences between regions at the beginning of a period, there will be migration of skilled workers. The ad hoc dynamics from the standard core-periphery model in Fujita et al. (1999) are assumed. Remaining analysis focuses on the steady state and any migration equation could be assumed to facilitate reaching the steady state. With zero transport costs there is no need to define real wages separately from nominal wages for the purpose of migration, because there are no price differences between regions. Firms may face different costs of innovation in different locations.

As such, a region with higher innovation costs (i.e. less knowledge spillovers available) will be unable to pay as highly for skilled labour to maintain profitability. This would lead to short-term wage differences that disappear with migration. Migration leads to the steady state and provides the requirement that skilled wages equalise between locations in the long run.

Technology

Production of an individual variety involves a fixed (labour) investment in a quality improving innovation (in the previous period) and a constant marginal cost. Production of each variety is contestable through these quality improvements produced by a competitive research and development sector with free entry. In each period, the quality leader produces variety i, j and potential investors or firms, including incumbents, choose whether to enter in the following period. If a firm enters, it selects a variety and conducts research effort to develop a quality improvement sufficient to gain a niche monopoly position for production in the following period only. Incumbent firms cannot retain the inter-temporal spillover beyond one period of production because free entry in the R&D sector allows each variety to be contestable. Therefore all production firms must invest in a quality improving innovation in the period prior to production and each firm's decision horizon is only one period long. Firms last for as long as they continue to innovate, but it is also possible to consider that each firm dissolves after one period of research and one period of production, to be replaced by a new innovating firm.

Multiple industries and varieties enables a multi-sector knowledge spillover mechanism to be added to the model of endogenous growth without scale assumptions (Young, 1998; Bond-Smith et al., 2018). The fixed cost of manufacturing in the subsequent period t is the skilled labour requirement in the previous period, $t-1$, to achieve the targeted quality level $A_{i,j,t}$ given by:

$$F_{i,j,t-1}(A_{i,j,t}, \bar{A}_{i,j,t-1}) = \begin{cases} \gamma e^{\eta A_{i,j,t}/\overline{A}_{i,j,t-1}} & \text{if } A_{i,j,t} \geq \overline{A}_{i,j,t-1} \\ \gamma e^{\eta} & \text{otherwise,} \end{cases} \tag{5}$$

where γ and η are constants that may be used for calibration and $\overline{A}_{i,j,t-1}$ is an index of technological opportunity for variety i, j, representing the intertemporal spillover of knowledge available to variety i, j researchers. The fixed cost can be thought of as two components: a standard fixed cost of γe^{η} irrespective of quality improvement and a cost of $\gamma e^{\eta A_{i,t}/\bar{A}_{i,j,t-1}} - \gamma e^{\eta}$ for achieving a quality improvement.

The spillover of knowledge between firms is imperfect. $\lambda_R < 1$ is a scalar that describes the proportion of knowledge that is available to a firm that is spatially separated from the location of that knowledge. The same logic of firms being separated by geographic space, can also be applied to manufacturers of different varieties being separated by technological space. Each component of knowledge is assumed to be weighted according to a related variety approach (Boschma & Frenken, 2009), where the relatedness of technology describes how useful the knowledge is to innovation in a firm's own variety (i.e. proximity in technological space). For simplicity, it is assumed that varieties in the same sector are weighted equally and varieties in other sectors are also weighted equally. Knowledge of a firm's own variety is given a weight of one, knowledge from innovations within the firm's own sector a weight of $\lambda_V < 1$ and knowledge of other sectors a weight of $\lambda_M < 1$, such that $1 > \lambda_V > \lambda_M \geq 0$. If the firm's selected variety was previously produced in the foreign region, it has the same spatial weight as any foreign knowledge $\lambda_R \leq 1$. Evidently, the relatedness of different varieties in the real world is not as simple. To reflect this, it is possible to weight knowledge from every individual pair of varieties by some kind of proximity measure from which firms choose an optimal location (Boschma, 2005) but this would not demonstrate additional insight in a theoretical model of growth. This technical externality in research and development triggers a "clustering effect" as it induces firms to cluster in locations alongside other firms in their own sector. Firms must also consider this effect alongside incentives to locate in a larger agglomeration where there are more sources of knowledge spillovers from other sectors. This incentive to locate with the larger share of manufacturing, is described as the "agglomeration effect".

The knowledge input to innovation is therefore made up of three components: knowledge of the variety's own quality level, knowledge from within the firm's own sector and knowledge from other varieties in other sectors. It is assumed that knowledge from all sources is additive. For developing a quality improvement to produce in period t, the knowledge spillover that is an input to innovation has three weighted components:

1. the knowledge at time $t-1$ from the firm's own variety i, j, represented the by quality level
$$A_{i,j,t-1} \text{ or } \widetilde{A}_{i,j,t-1}, \tag{5a}$$
2. the weighted average knowledge of quality from varieties within the firm's own sector i weighted by location
$$\overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} = \frac{\sum_{j\in n_i} A_{i,j,t-1} + \lambda_R \sum_{k\in\tilde{n}_i} \widetilde{A}_{i,k,t-1}}{n_i + \tilde{n}_i} \quad \text{and} \tag{5b}$$
3. a weighted average knowledge of quality improvement from other manufacturing sectors weighted by location
$$\overline{A}_{i\forall M,t-1} = \frac{\sum_{m\in M,\, j\in n_m} A_{m,j,t-1} + \lambda_R \sum_{m\in M,\, k\in\tilde{n}_m} \widetilde{A}_{m,k,t-1}}{\sum_{m=1}^{M} n_m + \sum_{m=1}^{M} \tilde{n}_m}, \tag{5c}$$

where A describes the quality improvement in each period and λ_R represents the weighting for knowledge that is sourced from firms in a different location than the firm producing variety i, j. Note that in the steady state with zero transport costs, the firms in each sector are clustered in either the home region or the foreign region, but not both. Therefore, each of these components will include only the home region variables or the foreign region variables. In an unsteady state, or between steady states, both types of variables could be included.

For a home region firm producing variety i, j, the overall index of technological opportunity is given by:

$$\overline{A}_{i,j,t-1} = \max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) + \lambda_V \overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}. \quad (5d)$$

That is, the index of technological opportunity is the knowledge associated with the latest innovations in the firm's own variety weighted by location plus a weighted average of the knowledge associated with innovations of all other varieties weighted by location and technological relatedness. As a result, firms may face a trade-off between the costs of innovation by locating in a cluster of technologically related firms or locating in an agglomeration of relatively unrelated firms. It is this trade-off which leads to the possibility of including clusters in an endogenous growth model and offers amenable implications for regional innovation growth policy.

It is assumed that the number of sectors is fixed such that there are always M sectors. The number of varieties in each sector is determined by the parameters of the model and new varieties can emerge to replace existing varieties. If the variety has never been produced before, the knowledge of a firm's own innovations is replaced by a weighted average of innovations for its selected sector i. This maintains symmetry in each sector even when a new variety is introduced. The index of technological opportunity for new varieties is given by:

$$\begin{aligned}\overline{A}_{i,j,t-1} &= \frac{\sum_{j\in n_i} A_{i,j,t-1} + \lambda_R \sum_{k\in\tilde{n}_i} \widetilde{A}_{i,k,t-1}}{n_i+\tilde{n}_i} + \lambda_V \overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1} \\ &= (1+\lambda_V)\,\overline{A}_{i\forall(n_i+\widetilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}.\end{aligned} \quad (5e)$$

This specification means that each firm in the same location faces the same costs of improving an existing variety in that region or introducing a new variety. No two firms choose the same variety because monopoly profits are always greater than individual duopoly profits. Analogous equations exist for foreign firms.

Short-run equilibrium

The short-run equilibrium describes the prices, wages, production and innovation in each period. Equilibrium prices, wages and production follow from optimisation within each manufacturing sector from Dixit-Stiglitz utility. The overall optimisation requires firms to choose a location, sector, variety and investment in quality-improving innovations based on the skilled worker location, consumers' taste for variety and quality and the firms' access to knowledge in alternative locations.

Prices, wages and production.

Consumers allocate expenditure across all sectors to maximise utility subject to the budget constraint

$$P_T C_T + \sum_{i \epsilon M} P_i C_i \leq E + \tilde{E}, \tag{6}$$

where $E+\tilde{E}$ describes world expenditure (which is normalised to one), P_T is the price of traditional goods, P_i is the price index of each manufacturing sector i and C_i is the total consumption index of each sector i given by Equation 1b. By optimisation and the nature of Cobb-Douglas preferences, $1-\mu$ is the share of expenditure spent on traditional goods and $\frac{\mu}{M}$ is the share spent on all varieties in each industrial sector i:

$$P_T C_T = (1-\mu)\left(E+\tilde{E}\right), \qquad P_i C_i = \frac{\mu}{M}\left(E+\tilde{E}\right) \quad \forall i \epsilon M. \tag{6a}$$

Traditional goods We include a 'traditional good' sector produced by immobile labour to elegantly model of the general equilibrium and steady state spatial pattern. This assumption allows for a more realistic spatial pattern that includes some level of immobile demand. The traditional goods sector is addressed briefly and otherwise the remainder of the analysis focuses on the knowledge sectors of innovation and manufacturing.

The traditional goods sector is perfectly competitive and has constant returns to scale. Unskilled workers provide one unit of production per period and with 1:1 technology, total production of traditional goods is $C_T = 1-\mu$ produced by both regions. Free trade of goods ensures equal nominal prices and wages in the two regions. With full employment of $1-\mu$ unskilled workers, 1:1 technology and normalised world expenditure $E+\tilde{E}=1$, the first part of Equation 6a is solved such that the traditional goods sector is the numéraire:

$$\begin{aligned} &w_T C_T = w_T(1-\mu) = P_T C_T = P_T(1-\mu) = (1-\mu)\left(E+\tilde{E}\right) \qquad \text{and} \\ &\tilde{w}_T C_T = \tilde{w}_T(1-\mu) = \tilde{P}_T C_T = \tilde{P}_T(1-\mu) = (1-\mu)\left(E+\tilde{E}\right) \end{aligned} \tag{6b}$$

Hence:

$$w_T = \tilde{w}_T = P_T = \tilde{P}_T = E+\tilde{E} = 1. \tag{6c}$$

Manufacturing Following a fixed investment in period $t-1$, firms may produce any quantity in period t at a constant marginal (labour) cost of $w_K\beta$. All workers provide one unit of labour per period. To simplify the model, it is assumed that only the quality leading firm in each variety can produce each period and any former incumbents exit the market such that all firms producing in the current period were innovators in the previous period. Alternative assumptions could also provide for monopoly procing such as the assumption in Akcigit & Kerr (2018). Moreover, there is no economies of scope and each variety is produced by a different manufacturer.

The Cobb-Douglas demand function from home region consumers for variety i, j is:

$$c_{i,j} = \frac{\mu\left(E+\tilde{E}\right)}{M} A_{i,j}^{\sigma-1} p_{i,j}^{-\sigma} P_i^{\sigma-1}, \quad (7)$$

where $p_{i,j}$ and $A_{i,j}$ represent the price and quality of variety i, j respectively and P_i is an index for sector i of the price and quality of all varieties in that sector given by:

$$\begin{aligned} P_i &= \left[\sum_{k=1}^{n_i+\tilde{n}_i} A_{i,k}^{\sigma-1} p_{i,k}^{1-\sigma}\right]^{1-\sigma} \quad (7a) \\ &= \left[n_i A_i^{\sigma-1} p_i^{1-\sigma} + \tilde{n}_i \tilde{A}_i^{\sigma-1} \tilde{p}_i^{1-\sigma}\right]^{1-\sigma}. \end{aligned}$$

The sector price index is defined such that the total amount spent on sector i is $P_i C_i = \frac{\mu}{M}\left(E+\tilde{E}\right)$ as in Equation 6a. While expenditure was normalised to one in each period, $E+\tilde{E}$, is retained for explanatory purposes.

In period $t-1$, each firm selects a quality improvement for production in period t with its associated cost of innovation (Equation 5) and its period t price to maximise the monopolistically competitive profits discounted for time:

$$\max_{p_{i,j,t}, A_{i,j,t}} \frac{(p_{i,j,t} - \beta w_{K,t})\, c_{i,j,t}}{1+r} - w_{K,t-1} F_{i,j,t-1}\left(A_{i,j,t}, \overline{A}_{i,j,t-1}\right), \quad (8)$$

where c_i is demand from domestic and foreign consumers for variety i produced in the home region, β is the marginal and per unit skilled labour requirement of producing one more unit of variety i, j. $F_{i,j,t-1}$ is the number of skilled workers required by the firm in the research and development sector to achieve a target quality level of $A_{i,j,t}$. Each firm maximises profit subject to the demand function for the domestic and exported variety that the firm produces. A home region manufacturer is subject to the demand function given by Equation 7. As innovation occurs in the period prior to production, firms discount future profit for time. It is assumed that each firm takes expected price and quality setting behaviour of other firms as given and ignores the effects of its own pricing decisions on the price quality index. That is, P_i (or $\tilde{P}_i$) is treated as fixed when differentiating by p_i. These assumptions are plausible with a sufficiently large number of firms in each sector. By differentiation, the first order conditions are given by:

$$c_{i,j,t} + (p_{i,j,t} - \beta w_{K,t}) \frac{\partial c_{i,j,t}}{\partial p_{i,j,t}} = 0, \quad (8a)$$

$$\frac{(p_{i,j,t} - \beta w_{K,t})}{1+r} \frac{\partial c_{i,j,t}}{\partial A_{i,j,t}} - w_{K,t-1} \frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}} = 0. \quad (8b)$$

As is usual in Dixit-Stiglitz models, free entry is assumed and the number of varieties in each sector is determined endogenously by the elasticity of substitution and the size of each region. With free entry into the research and development sector, firms make zero profits as its entire mark up is invested in innovation. Profits are zero in equilibrium, because if profits were positive, the marginal skilled worker could shift to the R&D sector and produce greater quality improvements or an additional variety. The free entry condition is given by:

$$\frac{(p_{i,j,t} - \beta w_{K,t})\, c_{i,j,t}}{1+r} = w_{K,t-1} F_{i,j,t-1}\left(A_{i,j,t}, \overline{A}_{i,j,t-1}\right). \tag{8c}$$

From the first-order conditions, rearranging Equation 8a finds that prices are a constant mark-up over marginal cost based on the elasticity of substitution:

$$p_{i,j,t} = \frac{\sigma}{\sigma-1}\beta w_{K,t}. \tag{9}$$

With zero transport costs, exported varieties have the same prices as domestic varieties. It is also possible to normalise manufacturing prices to one by setting $\beta = \frac{\sigma-1}{\sigma}$, but in the model here, β remains to describe labour productivity.

Innovation

Dividing the second first order condition (Equation 8b) by the free entry condition (Equation 8c) and rearranging, firms select a quality improvement where the elasticity of research cost with respect to quality is equal to the elasticity of demand with respect to quality.

$$\begin{aligned} \varepsilon^{c_{i,j,t}}_{A_{i,j,t}} &= \varepsilon^{F_{i,j,t}}_{A_{i,j,t}} \\ \sigma - 1 &= \eta \frac{A_{i,j,t}}{\overline{A}_{i,j,t-1}}. \end{aligned} \tag{10}$$

Rearranging Equation 10 obtains:

$$\frac{\sigma-1}{\eta} = \frac{A_{i,j,t}}{\overline{A}_{i,j,t-1}}, \tag{10a}$$

which describes the preference of firms to invest in quality improvement. By substitution into Equation 5, the cost of innovation or preference to invest and the number of skilled workers employed in research by each firm per period is:

$$F_{i,j,t} = \gamma e^{\frac{\eta A_{i,j,t}}{\overline{A}_{i,j,t-1}}} = \gamma e^{\sigma-1}. \tag{10b}$$

Rearranging further, firms select a quality target of:

$$A_{i,j,t} = \frac{\sigma-1}{\eta}\left[\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) + \lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}\right]. \tag{10c}$$

This is a quality improvement multiplier of:

$$\frac{A_{i,j,t}}{\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right)} = \frac{\sigma-1}{\eta}\left[1 + \frac{\lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}}{\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right)}\right]. \tag{10d}$$

Assuming this multiplier is always greater than one, there are always quality improvements in equilibrium.

Quality improvement per period is given by:

$$I_{i,j,t} = A_{i,j,t} - \max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) =$$
$$\left(\frac{\sigma-1}{\eta}-1\right)\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right)+\frac{\sigma-1}{\eta}\left[\lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}\right]. \tag{11}$$

Intuitively Equation 11 (also Equations 10c and 10d) has two components. Quality improvement is made up of the innovation from direct investment in R&D (or R&D-based innovation):

$$\left(\frac{\sigma-1}{\eta}-1\right)\max\left(A_{i,j,t-1}, \lambda_R \widetilde{A}_{i,j,t-1}\right) \tag{11a}$$

plus the quality improvement due to the variety specific knowledge spillover:

$$\frac{\sigma-1}{\eta}\left[\lambda_V \overline{A}_{i\forall(n_i+\tilde{n}_i),t-1} + \lambda_M \overline{A}_{i\forall M,t-1}\right] \tag{11b}$$

giving the total quality improvement as given in Equation 11.

Labour market clearing and endogenous variety

Labour market clearing requires that the total labour used in home region manufacturing (L_M) and R&D (L_R) are equal to the total supply of regional skilled workers (L_K). In equilibrium, the skilled labour used in manufacturing in the home region is the worldwide expenditure on manufactured goods produced in the home region divided by the price per unit and multiplied by its marginal cost:

$$L_M = \frac{\mu S\left(E+\tilde{E}\right)}{p}\beta = \frac{\sigma-1}{\sigma}\mu S\left(E+\tilde{E}\right), \tag{12}$$

where $S = \frac{\sum_{i\epsilon M} n_i p_i c_{i,}}{\sum_{i=1}^{M} n_i p_i c_{i,} + \sum_{\tilde{i}=1}^{M} n_i \tilde{p}_i \tilde{c}_i} = \frac{\sum_{i\epsilon M} n_i p_i c_{i,}}{\mu\left(E+\tilde{E}\right)}$ is the total market share of manufacturing expenditure held by home region firms. The labour used in research is equal to the number of firms in the next period multiplied by the investment in research labour by each individual firm:

$$L_{R,t} = \gamma e^{\sigma-1}\sum_{i=1}^{M} n_{i,t+1}. \tag{13}$$

Home region skilled labour market clearing in period t therefore requires:

$$L_{K,t} = \frac{\sigma-1}{\sigma}\mu S\left(E+\tilde{E}\right) + \gamma e^{\sigma-1}\sum_{i=1}^{M} n_{i,t+1}. \tag{14}$$

Analogous equations exist for foreign region manufacturers.

The reward for investing in R&D to develop a quality improvement is the operating profit in the following period. With constant mark-up over marginal cost, the operating profit π is the value of sales shared equally between firms, divided by σ: $\pi = \frac{1}{\sum_{i=1}^{M} n_i} \frac{\mu S\left(E+\tilde{E}\right)}{\sigma}$. By the free entry condition (Equation 8c), the amount each firm spends on innovation is its present value share of profit. The free entry relation can be rearranged to:

$$\pi_t = \frac{\frac{\sigma-1}{\sigma}\mu S\left(E+\tilde{E}\right)}{\sigma \sum_{i=1}^{M} n_{i,t}} \frac{1}{(1+r)} = \gamma e^{\sigma-1} = F_{t-1}. \tag{15}$$

Substituting this expression (advanced one period) into Equation 14, obtains:

$$L_{K,t} = \frac{(\sigma-1)}{\sigma}\mu S_t\left(E_t+\tilde{E}_t\right) + \frac{\mu S_{t+1}\left(E_{t+1}+\tilde{E}_{t+1}\right)}{\sigma(1+r)}. \tag{15a}$$

Rearranging the Euler equation given in Equation 2 and solving for the value of consumer expenditure on manufactured goods as a function of the parameters gives:

$$\mu S_t\left(E_t+\tilde{E}_t\right) = \frac{\sigma L_{K,t}}{(\sigma-1)+\alpha}. \tag{15b}$$

Equilibrium requires the economy to move to a steady state level of consumer expenditure with a constant interest rate $1+r = 1+\rho = \frac{1}{\alpha}$. Substituting (Equation 15b) into the modified free entry relation (Equation 15) and rearranging determines the total number of firms in the home region across all industries as a function of the parameters:

$$\sum_{i=1}^{M} n_i = \frac{L_K \alpha}{[(\sigma-1)+\alpha]\,\gamma e^{\sigma-1}}. \tag{15c}$$

In the steady state an analogous function exists for foreign firms. Similarly, the total number of firms in each industry in the steady state across both regions is given by:

$$n_i + n_{\tilde{i}} = \frac{\left(L_K+\tilde{L}_K\right)\alpha}{M[(\sigma-1)+\alpha]\,\gamma e^{\sigma-1}} = \frac{\mu\alpha}{M[(\sigma-1)+\alpha]\,\gamma e^{\sigma-1}}. \tag{15d}$$

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