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The decades-long dispute over scale effects in the theory of economic growth

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Abstract

The so-called 'new growth theory' is characterized by the now Nobel Prize winning insight that ideas are a non-rival input to and output from endogenous investment in innovation. Non-rivalry implies increasing returns to scale, but this also unintentionally creates an empirically-disputed scale effect that a growing population implies an ever-increasing growth rate. Empirical evidence supports fully-endogenous growth without scale effects, but theoretical issues sustain the decades-long dispute over exactly how to negate the scale effect. This article surveys theoretical approaches to resolving the scale effect and shows how four generations of endogenous growth theory are defined by the maturing of modeling techniques for constraining increasing returns. The synthesis suggests that the dispute over scale effects is really a narrative about how the powerful application of increasing returns has followed a standard theoretical development pattern. This implies that a fourth generation is now emerging that negates the scale effect while retaining fully-endogenous growth without relying on assumptions of linearity. Instead, the market response to excessive increasing returns to innovation constrains explosive growth by expanding the market, rather than by a linear assumption. This latest class of endogenous growth models may be the final chapter to resolving the long running dispute.

JEL Classifications: E10; L16; O41
Keywords: endogenous growth; scale effects; increasing returns; innovation; invention.

1 Introduction

The so-called *'new growth theory'* began with Paul Romer's (1990) now Nobel Prize winning insight that ideas are a non-rival input to and output from investment in innovation in return for temporary monopoly profits. Non-rivalry implies increasing returns to scale, which is fundamental to endogenizing growth, because productivity depends only on the stock of ideas rather than its division between people. Increasing returns enabled Romer (1990) to endogenise research and development into long run models of growth. This means that the incentives for investing in technological change come from economic factors and economic policies can influence the rate of growth by changing the incentives faced by those developing new technologies. In particular, endogenous growth theory implies that increased research effort leads to a faster rate of idea discovery and a higher rate of growth. Increasing returns is also the basis for many other intuitive results in economics, but increasing returns appears in new growth theory in very a unique way: the scale of production of *ideas*.

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However, this also implies the scale effect: that a growing population leads to explosive growth due to constant growth in the number of researchers making use of the same non-rival ideas. The first generation of endogenous growth models (Romer, 1986, 1990; Grossman & Helpman, 1991; Aghion & Howitt, 1992) are all characterized by this unintended and empirically-disputed (Jones, 1995b) scale effect. Subsequent theorists have debated for decades over the appropriate modeling techniques to remove such scale effects while retaining endogenous innovation to reflect economic growth in the real world. The latest Nobel Prize in economics brings an opportunity to reflect on how growth theory has progressed to incorporate the concepts of non-rival ideas and increasing returns within theoretical models that reflect empirical realities.

This article surveys the key contributions in the debate about scale effects in the theory of economic growth by focusing on the particular techniques used to negate explosive growth. The synthesis here shows how the dispute over scale effects can be characterised by a repeated developmental pattern that now applies solutions that solve for equilibrium under increasing returns to the concept of producing ideas. In this way, the dispute over scale effects is really a narrative about the powerful and useful nature of increasing returns to scale and the intellectual challenge to solve its paradoxical equilibrium.

Jones (1995b) identified that data showed constant growth over time despite a growing population. In response, Jones (1995a), Kortum (1997) and Segerstrom (1998) argue that scale effects should be removed by diminishing the marginal productivity of cumulative research effort such that ideas become 'harder to find'. These models assume diminishing returns to the scale of cumulative ideas as a kind of modeling trick that diminishes the impact of increasing returns due to population growth, until there is constant returns to scale for innovation in the balanced growth path. In this way, the properties of the balanced growth path do not require a linear assumption to prevent explosive growth. However, Jones (1995a) describes these second generation models as *'semi-endogenous'*, because long run growth rates are unaffected by research effort and only sustained by population growth even though innovation is an equilibrium response of profit maximizing agents. Nordhaus (1969) also noted that population growth might be required to sustain technological improvement. Alternatively, Young (1998), Dinopoulos & Thompson (1998), Peretto (1998) and Howitt (1999) contend that allowing the number of varieties to expand with population negates the aggregate scale effect by spreading growing research effort across more varieties, while retaining 'fully-endogenous' growth for quality improving innovations. In this way, the aggregate scale of increasing returns for innovation is diminished by the entry of new varieties. These *'Schumpeterian'* growth models contend that increased innovation effort does indeed increase the long run growth rate through improvements in industry productivity. Finally, in what this article argues to be the *fourth generation* (4G), Peretto (2018) allows for ideas to be explosive with 'excessive' increasing returns, but growth is constrained because ideas expand in two dimensions.

There are many ways in which each generation of models can be identified as distinct. The synthesis here identifies the development of modeling techniques along three conceptual themes. Firstly, the different approaches used to resolve the scale effect are essentially repeated applications of various techniques to solve for equilibrium under increasing returns in a different context. For example, in much the same way as free entry and imperfect competition constrains the impact of increasing returns on prices, the entry of variety-expanding ideas constrains the impact of increasing returns on innovation. Secondly, semi-endogneous and 4G models differ from first generation and Schumpeterian models, because constant growth is a characteristic of the balanced growth path rather than restrictive assumptions about the functional form of technological change.[1] Thirdly, the different approaches vary by the extent that such restrictive assumptions remain to negate the scale effect if the particular entry mechanism is insufficient. Given the absence of any linear assumptions at all in Peretto (2018), the dispute over scale effects in the theory of endogenous growth may be finally reaching a conclusion.

The article is closely related to other attempts to reconcile the various approaches to remove scale effects in the endogenous growth literature. Jones (2002) suggests that rising research intensity and education levels can sustain higher constant growth rates temporarily, driving a large portion of recent U.S. growth, while long run growth is still only dependent on population growth. On the other hand De Long & Summers (1991), Mankiw et al. (1992) and Young (1995) contend that recent growth trends may be more related to diminishing returns to capital, in all its forms, rather than the nature of innovation. Similarly, Gordon (2018) and Madsen (2018) identify that a number of transitional events explain higher rates of growth since the industrial revolution such that long run growth rates could be expected to diminish, whether growth is semi- or fully-endogenous. Jones & Romer (2010) acknowledge that first generation and semi-endogenous models both closely match stylized facts about growth over relatively long periods such as a century. The hybrid models in Cozzi (2017a,b) propose that technological improvement can be caused by both semi- and fully- endogenous components. However, even when growth is minimally caused by the semi-endogenous element, this factor dominates the long run such that growth is only fully-endogenous if population growth is negative or shrinking (Cozzi, 2017b). Alternatively, combining semi-endogenous and Schumpeterian growth using a CES aggregator suggests the dominance of one or the other depends upon whether semi-endogenous or Schumperterian growth are complements or substitutes (Cozzi, 2017a). This article takes a different approach by recognizing that the proposed solutions to remove the scale effect follow a standard theoretical development pattern for reconciling increasing returns at the firm level and negating their impact in aggregate.

The article makes five distinct contributions to the literature. Firstly, Section 2 redefines the scale effect problem in terms of increasing returns and explosiveness in the development of endogenous growth theory. Section 3 surveys key theoretical contributions to the decades-long dispute about how to correctly model endogenous growth without scale effects while retaining increasing returns. This synthesis brings together the various strands of theoretical research to shed light on the similarities and differences between the various approaches and assumptions that define four generations of theoretical models. In doing so, the third contribution of the paper becomes clear by identifying and defining the latest generation of models that is emerging. In particular, mechanisms respond to excessive increasing returns to achieve linearity in the steady state and only in the steady state, not by assumption. Such mechanisms negate explosive ideas, the scale effect and the linearity critique. While the focus of this article is on theoretical methods, Section 4 briefly surveys the empirical evidence. Lastly, discussion of the dispute implies that this fourth generation has finally resolved the decades long dispute. This sends research in an all new direction to examine how the market responds to excessive returns to innovation allowing explosive ideas but constraining explosive growth. Lastly, Section 6 offers some concluding thoughts for future research.

2 The scale effect, increasing returns and explosiveness in growth theory

To understand the scale effect and its proposed solutions, it is helpful to first consider the development of new growth theory. The simplest model of growth assumes perfect competition and growth by capital accumulation, but growth is unsustainable if capital faces diminishing returns. In response, Solow (1956) augments capital with improvements in exogenous technology such that growth is sustained solely through improvements in Total Factor Productivity (TFP). That is, technological improvement increases production, inducing investment and an expanding capital stock. While growth accounting attributes a share of growth to capital deepening, all capital deepening in the steady state is induced by TFP growth so all output growth is a result of TFP growth. On this basis, growth theories focus on technological change as the engine of growth. But Solow (1956) offers no explanation for technological improvement. Romer (1990) explained that technological improvement was the result of profit maximizing agents investing in research effort in return for temporary monopoly profits. Romer (1990) emphasizes that knowledge spillovers allow researchers to benefit from the existing stock of *non-rival* ideas as inspiration for discovering new non-rival ideas.

Non-rivalry enables increasing returns to scale, because only one version of the original idea is required to increase output. For example, a business can double its output by replicating its factory. The factory design or idea can be used to build many factories so is considered non-rivalrous. But a business can *more* than double its output if its new factory uses a new design. There is constant returns to scale in the rivalrous factors of production, such as labor and capital, but there is increasing returns to scale in the rivalrous and non-rivalrous factors *combined*.

Increasing returns is a much older and broader problem for economic theory, because it is incompatible with perfect competition. The puzzle of finding equilibrium in theoretical models with increasing returns is a paradox of economic theory that dates back to Adam Smith's division of labor. Among other things, increasing returns in new growth theory implies that the price of an idea is not constrained by its marginal product. Romer (1990) is not the first to notice the process of invention, the accumulation of knowledge or the role of knowledge spillovers, but technological change with increasing returns to scale was difficult to formalize in earlier models. As a result, growth models had left technological change effectively exogenous.

For example, Arrow (1962) argued that new knowledge is the result of learning by doing, with discoveries emerging due to industry wide cumulative investment and production. Nordhaus (1969) sketches a model of invention in which the cost of invention extracts the future surplus of patent rents, as in the free entry condition, but technology remained effectively exogenous for technical reasons. In sharing the Nobel prize with Paul Romer, William Nordhaus is recognised for his contribution to integrating climate change into long-run macroeconomic analysis, but this synthesis implies a much closer relationship with Paul Romer's work. However, William Nordhaus acknowledges himself that incorporating technological change into long run models of growth was at that stage "too hard".[2]

Avinash Dixit and Joseph Stiglitz's (1977) model of monopolistic competition provided the technical solution that enabled firms to earn monopoly profits, above marginal cost. This provides the incentive for investing in R&D. Krugman (1979, 1991) showed how this tool could be embedded into a static general equilibrium model. Romer (1986) showed how to solve the dynamic model. The last step was combining these tools with the concept of non-rival ideas in a single model. Romer (1990) formalizes the insights of earlier theorists using these new tools that deal with increasing returns in a dynamic general equilibrium model. Many of the fields that resolve this issue using the Dixit-Stiglitz (1977) model of monopolistic competition are also referred to as so-called '*new*' theories in economics. It is perhaps no surprise that some pioneers of new growth theory had previously worked on new trade theories (for example Helpman (1981); Helpman & Krugman (1985)).

Yet increasing returns is particularly unique in new growth theory, because ideas are the non-rival input *and output* from innovation. Increasing returns only requires an increase in the rival factors of production to proportionally increase output, but the non-rival factor is increasing endogenously. As a result, the concurrent flow of ideas and population growth continually increase the scale of research effort and the rate of technological improvement, implying a growth rate that increases to infinity in finite time. That is, constant growth in the rival factors of production implies explosive growth.[3] Such explosive growth is constrained in first generation models by the restrictive assumption of no population growth. The focus of this article is the dispute over the appropriate theoretical techniques to resolve this unique form of increasing returns without such restrictive assumptions.

However, this is not the only restrictive assumption that constrains explosive growth. Growth models typically require a critical 'knife-edge' assumption that at least one differential equation is linear to facilitate positive growth but avoid explosive growth (Jones, 1999, 2005). That is, there is some differential equation $\dot{X} = fX$ where X is a model specific variable that grows indefinitely in the steady state and f is effectively assumed constant with respect to X (Jones, 2005). Anything less than linear eventually results in zero growth while anything more than linear is explosive. The so-called 'linearity critique' is that such strict linear assumptions must be justified on more than just technical reasons (Jones, 1999, 2005).

Endogenous growth theories can be defined into generations by the linearity critique. First generation models make this linear assumption in the aggregate ideas production function. Semi-endogenous models take the logical approach of assuming linearity only within population growth. Schumpeterian models assume the ideas production function is linear at the industry level.

The critique is particularly directed at first-generation and Schumpeterian models because the assumption is made in the innovation equation itself, sustaining the debate about the appropriateness of such assumptions. The semi-endogenous growth model is appealing since increasing returns is retained in the production of ideas, but constant growth emerges as a property of the model in the balanced growth path. Yet semi-endogenous models deny the purpose of endogenizing innovation into growth models in the first place, requiring the semi- label, because per capita long run growth is unaffected by research effort.

The wealth of empirical evidence seems to support fully-endogenous Schumpeterian growth,[4] but the linearity critique fuels counteractive research in support of semi-endogenous growth[5] including recent studies showing that ideas are becoming 'harder to find' (Bloom et al., 2018). This on-going dispute between economists about how to correctly model endogenous growth when theoretical models are confronted with contradictory empirical evidence requires a resolution in order to influence effective real-world innovation and growth policy (Romer, 2015).

The way ideas build on each other does not necessarily need to be limited by linearity. Paul Romer describes that ideas are compounding: that the list of possible ideas does not just add up, it multiplies.[6] Linear assumptions are simply a modeling trick to allow a balanced growth path to be found. Romer (1994) is even explicit about this with respect to the linear specification in Romer (1990). In this way, explosiveness may not need to be entirely disallowed. The last piece of the puzzle that enables explosive ideas but prevents explosive growth and the scale effect is missing from Schumpeterian growth theory. On this basis, the market response approach developed in Peretto (2018) should be considered as a new fourth generation of endogenous growth theory. This class of models negates explosiveness without linear assumptions by allowing a market response to 'excessive' increasing returns and population growth. This allows explosive ideas but leads to the required linearity as an equilibrium outcome in the steady state and only in the steady state. Peretto (2018) proposes product proliferation as such a mechanism. As a result, explosive growth is constrained by the entry of new industries in response to both population growth and 'excessive' increasing returns.

3 A disputed theory

It would be impossible to review every theoretical contribution in the long-running debate about scale effects, let alone about the role of increasing returns in economic growth, but this section considers the crucial models that have developed since Romer (1990).

With the help of simple 'toy' models, this section briefly outlines four generations of R&D based growth models: (i) initial models of endogenous growth that imply a scale effect; (ii) semi-endogenous growth where scale effects are removed, but long run growth rates are unaffected by policies to support R&D; (iii) Schumpeterian endogenous growth models without scale effects that impose the linearity assumption with respect to firm-level technological improvement; and the latest (iv) 4G class of models in which a mechanism responds to excessive increasing returns and population growth to constrain explosive growth, not by linear assumption.

Consider an economy where final output is given by $Y_t = A_t^{\sigma} L_{Yt}$ where L_{Yt} is labor used in production, A_t represents the stock of knowledge or technological productivity and σ is a parameter for calibration. In order to focus on technological change as the sole engine of growth in the steady state, the role of physical capital and human capital is ignored. Research effort builds on the existing stock of knowledge to develop new ideas and increase productivity. There is free entry for entrepreneurs who can develop a new idea that builds on the knowledge stock. The exact micro-details of imperfect competition between entrepreneurs varies between models, but essentially price is constrained above its marginal product by imperfect competition and all profits from invention are used for research due to the free entry condition. The flow of new ideas is given by the function $\dot{A}_t = f\left(L_{At}, A_t\right)$ where L_{At} is labor devoted to research effort. Along the balanced growth path, a constant share of labor (s) is employed in research such that $L_{At} = sL_t$. Several alternative functions for $\dot{A}_t$ have been proposed.

3.1 First generation models of endogenous growth

Romer (1990) recognized the non-rivalry and accumulation of ideas in the innovation production function. While Romer (1990) assumes that growth comes with an increasing variety of designs, Grossman & Helpman (1991) and Aghion & Howitt (1992) develop similar models where growth comes in the form of productivity or quality improvements to a fixed number of intermediate varieties or sectors. Helpman (1992) shows how increasing variety and quality improvement are largely equivalent. While each model has differing micro-structures, these first generation models endogenize growth by firms investing in research effort to generate technological improvements that enable the firm to collect temporary monopoly profits as the incentive for investment.

In these first generation models, the function

$$\dot{A}_t = \gamma L_{At} A_t \tag{1}$$

describes how new ideas improve productivity over time where $\gamma > 0$ is a parameter for calibration. With labor market clearing, per capita output along the balanced growth path is given by $y_t = \frac{Y_t}{L_t} = A_t^{\sigma}\left(1 - s\right)$ such that per capita productivity is proportional to the stock of knowledge. Taking the time derivative using the chain rule and dividing by output yields the per capita growth rate

$$g_y = \frac{\dot{y}_t}{y_t} = \sigma\gamma L_{At} = \sigma\frac{\dot{A}_t}{A_t} = \sigma\gamma sL. \tag{2}$$

Constant research effort yields sustained growth, but population size implies the level of research effort due to a constant share of labor devoted to research. This generates a scale effect where the growth rates of technology ($g_A = \frac{\dot{A}_t}{A_t}$) and output are proportional to the size of the population $g_{At} = \gamma s L_t$. Intuitively, the scale effect is caused by a combined effect of a larger population increasing the supply of R&D workers that all make use of the same stock of cumulative non-rival ideas and increasing the size of the market that is captured by a successful innovator. This implies that a population growing at a rate of n results in a growth rate that increases to infinity in finite time ($g_{At} = \gamma s L_0 e^{nt}$). This is referred to as the '*scale effect*' in its 'strong' form, where long run per capita growth, is an increasing function of both the knowledge stock and the scale of the economy. The scale effect implies that larger economies grow faster than smaller ones and that population growth implies an ever-increasing (explosive) growth rate (See Figure 1).

[Figure 1 here]

3.2 Second generation, semi-endogenous growth models without scale effects

According to first generation theories, permanent changes in the potential determinants of long run growth should lead to permanent changes in long run growth. In critiquing first generation models, Jones (1995b) showed that TFP growth in the United States has remained constant, despite increasing investment in R&D and an increasing number of scientists and engineers. In response, Jones (1995a) proposed a second generation endogenous growth model without scale effects. In second generation models (Jones, 1995a; Kortum, 1997; Segerstrom, 1998) it is assumed that research becomes more difficult with cumulative innovation due to increasing technological sophistication. The initial appeal of this particular approach is that it explains why increased research effort over time due to population growth has not resulted in higher long run growth rates.

As with first generation models, each model has differing micro-structures and all assume that technological improvement comes from investment in research effort by profit-maximizing agents in order to acquire temporary monopoly profits. In contrast to first generation models, any increase in effort only results in a temporary increase in the growth rate as technological improvements are constantly eaten away by diminishing returns to the cumulative knowledge stock. As a result, these models imply that additional research effort only affects the level of income and has no affect on the long run growth rate, attracting the label '*semi*-endogenous'. Despite this result, the market failure that justifies support for innovation due to knowledge spillovers still applies but is measured in terms of level effects rather than changes in the long run growth rate. Therefore, semi-endogenous growth models still typically acknowledge a strong role for R&D policy to optimize research effort.

In these models, the function

$$\dot{A}_t = \gamma L_{At}^{\lambda} A_t^{\phi} \tag{3}$$

describes how productivity improves over time. Duplication implies diminishing returns to additional research effort at a point in time since some share of additional effort is wasted on tasks that are also performed by others. The parameter $0 < \lambda < 1$ allows for duplication in research effort where $\lambda = 1$ implies that no researchers duplicate the effort of other researchers and $\lambda = 0$ implies that all researchers duplicate the effort of other researchers (Jones, 1995a). R&D becomes progressively more difficult as the ideas that are easiest to discover are found first, implying diminishing returns to the knowledge stock at a point in time. The parameter $\phi > 0$ describes how research builds upon past knowledge discoveries, but $\phi < 1$ implies this increasing difficulty due to increasing technological sophistication. A recent empirical study suggests that ideas are more difficult to find over time (Bloom et al., 2018). $\gamma > 0$ is a parameter for calibration.

As in first generation models, labor market clearing requires that per capita output along the balanced growth path is given by $y_t = \frac{Y_t}{L_t} = A_t^{\sigma}(1-s)$ such that per capita productivity is proportional to the stock of knowledge. Taking the time derivative of per capita output and solving for the steady state per-capita growth rate finds that the long run per capita growth rate is proportional to the growth rate of knowledge. A stable balanced growth path requires that $\frac{\dot{y}_t}{y_t}$ is constant such that both $\dot{A}_t$ and A_t grow at the same rate. Rearranging the ideas production function to find the growth rate of technology, $g_A = \frac{\dot{A}_t}{A_t} = \gamma L_{At}^{\lambda} A_t^{\phi-1}$ and differentiating with respect to time, a balanced growth path where $\frac{\dot{A}_t}{A_t}$ is constant implies

$$g_A = \frac{\lambda n}{\gamma(1-\phi)} \tag{4}$$

and $g_y = \frac{\sigma \lambda n}{\gamma(1-\phi)}$, where n is the population growth rate. As a result, long run growth is unaffected by the proportion of labor devoted to research and only related to the population growth rate.

These models imply that the steady state productivity of cumulative knowledge diminishes over time (see Figure 2) but a growing population can sustain growth in per capita incomes. The parameter ϕ plays a critical role. If $\phi = 1$ the model reverts to the standard endogenous growth case, with some nuances. But if $\phi < 1$, growth diminishes until it is matched by the rate of population growth, leading to the balanced growth path. If $\phi > 1$ then growth is explosive, even without population growth. This specification also has implications for the predictions of endogenous growth models. For example, Aghion et al. (2017) finds that if artificial intelligence (AI) is able to produce new ideas then AI would have only transitory effects on growth if $\phi < 1$ but would imply greater long run growth rates if $\phi = 1$.

[Figure 2 here]

While research effort is endogenous, growth is constant, because the increasing scale of innovation effort is matched in the balanced growth path by diminishing returns to the extent of knowledge through $\phi < 1$. In much the same way as Solow's assumption of exogenous technological improvement mitigated diminishing returns to capital accumulation, population growth mitigates diminishing returns to cumulative knowledge. To see this, return to the intermediate steps in the product rule when differentiating the growth rate $g_A = \frac{\dot{A}_t}{A_t} = \gamma L_{At}^{\lambda} A_t^{\phi-1}$ with respect to time. Holding the stock of knowledge constant and differentiating with respect to time, the equation

$$\frac{\partial \left(g_{A,\dot{A}_t=0} \right)}{\partial t} = \gamma \frac{\dot{L}_t}{L_t} \left(\frac{\dot{A}_t}{A_t} \right) \tag{5}$$

describes how the growth rate increases over time due to increasing returns to the scale of research effort from population growth, as in the scale effect. Similarly, holding population constant and differentiating with respect to time, the equation

$$\frac{\partial \left(g_{A,\dot{L}_t=0} \right)}{\partial t} = (\phi - 1) \gamma \frac{\dot{A}_t}{A_t} \left(\frac{\dot{A}_t}{A_t} \right) \tag{6}$$

describes how the growth rate diminishes over time due to increasing difficulty until growth would be eventually zero in the balanced growth path, since $\phi < 1$. Taken together, the diminishing returns assumption effectively nullifies the compounding nature of the scale effect. Reasserting that both the knowledge stock and population grow over time, the balanced growth path implies $\frac{\partial (g_A)}{\partial t} = \frac{\partial \left(g_{A,\dot{A}_t=0} \right)}{\partial t} + \frac{\partial \left(g_{A,\dot{L}_t=0} \right)}{\partial t} = 0$, which can be easily solved for the constant growth rate $g_A = \frac{\lambda n}{\gamma(1-\phi)}$.

The semi-endogenous approach is appealing, because the innovation function is not linear yet constant growth emerges as a property of the balanced growth path through the interaction between increasing and diminishing returns to increasing scale and cumulative knowledge respectively. Linearity is still assumed in population growth but is not assumed in the critical growth equation. Li (2000) followed these models with a combination of both product variety and quality ladders dimensions of R&D that can generate fully-endogenous growth without scale effects but showed that if there are spillovers between the horizontal and vertical dimensions of R&D then such a model still generates semi-endogenous growth.

The scale effect is not removed entirely. Semi-endogenous growth models still have what Jones (2005) describes as 'weak' scale effects. In the balanced growth path, growth rates are unaffected by scale, but the size of an economy exhibits increasing returns to scale affecting per capita incomes. Jones (2005) argues that such an assumption is merely a characteristic of the non-rival input to innovation: existing ideas. Yet economists should also strive to understand the mechanisms that might cause such income effects, rather than rely on almost arbitrary side-effects of assumptions. Such assumptions seem almost lazy, raising more questions than they answer, because where do you draw the boundaries to test such predictions in a world of multi-national corporations and global supply chains? Even Jones (2005) recognizes that evidence of any weak form of scale effects can only be found in special case, approximated experiments. Recognizing such limitations, recent proposals suggest that removing scale assumptions altogether is important for modeling regional growth such that practical mechanisms can be used to explain and understand the source of any observed scale effects at the regional level, such as sources of agglomeration economies (Bond-Smith et al., 2018).

In addition to the linearity critique, political ideals of small government may also fuel support for semi-endogenous growth theories by misinterpreting the implications of the conclusion that support for R&D has no effect on long run growth rates. The market failure that arises from the non-rival nature of knowledge is still present in these models. For example, Jones & Williams (1998) use a semi-endogenous model to show that optimal R&D investment is at least two to four times actual investment. Instead, support for R&D has transitional impacts on growth rates and level effects on incomes and welfare. Jones (1997) finds that support for R&D could even diminish the long-run growth rate, but describes that support is still welfare improving because of the level effects. The implication that governments shouldn't support R&D is not a characteristic of the debate about how to remove scale effects. Instead the scale effects dispute relates only to what is observed as a result of such support.

3.3 Schumpeterian endogenous growth models without scale effects

Alternatively, to prevent explosive growth, Young (1998) recognized that a larger population leads to a larger number of firms and a wider variety of products, but growth is ultimately related to the growth rate of firm-specific quality. These so-called 'Schumpeterian' endogenous growth models negate the scale effect by recognizing two dimensions of innovation, quality improvements and new varieties (Young, 1998; Dinopoulos & Thompson, 1998; Peretto, 1998). This allows variety to expand in proportion to population growth and retains endogenous quality improvements as the engine of growth. While the assumptions of each model differ, implying various modeling limitations, each model retains fully-endogenous growth in firm or sector level productivity such that the long run growth rate can be influenced by research effort. While growth in Young (1998) is not influenced by a proportional R&D subsidy, it is still considered fully-endogenous, because long run growth is influenced by innovation effort through alternative R&D support policies that either focus solely on quality improvements (Dinopoulos & Thompson, 1999) or on the intensity of R&D (Young, 1998).[7]

To retain the stylized fact that long run growth is influenced by proportional support for research effort, (Howitt, 1999) adds an assumption that it becomes progressively more difficult to develop additional varieties for larger populations. This insight also helps to explain declining innovation productivity as observed by Jones (1995b) while retaining fully-endogenous growth.

In the Schumpeterian class of models, *firm level* innovation is the important function. Output by firm i is given by $Y_{it} = A_{it}^{\sigma} L_{Yit}$ with aggregate economic output given by $Y_t = \left[\int_0^{F_t} Y_{it}^{\frac{1}{\alpha}} di\right]^{\alpha}$ where $F = \eta L$ represents the number of varieties at time t, η is constant and α describes consumers' love of variety. In these models, the function

$$\dot{A_{it}} = \gamma L_{Ait} \overline{A_{it}} \tag{7}$$

describes firm level innovation in technology improvements, where $\gamma > 0$ is a parameter for calibration and $\overline{A_{it}} = \int_0^{F_t} \frac{A_{jt}}{F_t} dj$ represents economy-wide productivity. Labor market clearing requires $\int_0^{F_t} (L_{Yit} + L_{Ait})\, di = L$. Following Laincz & Peretto (2006) the symmetric equilibrium can be used to understand aggregate productivity such that economy wide output can be represented by $Y_t = F_t^{\alpha} A_t^{\sigma} L_{Yt}$ where A and L now represent firm averages. As a result, average productivity improves according to $\dot{A}_t = \gamma L_{At} A_t$ such that the functional form of first generation models applies at the firm level. Aggregate output per capita is given by $y_t = \frac{Y_t}{L_t} = F_t^{\alpha-1} A_t^{\sigma} (1-s)$. Differentiating with respect to time finds that growth is dependent upon research effort at the firm or sector level. Similarly, rearranging the flow of ideas finds that the growth rate of technology is

$$g_{At} = \frac{\dot{A}_t}{A_t} = \gamma L_{At}. \tag{8}$$

Similarly, the growth rate of per capita output is a function of the growth rate of technology at the firm level, $\frac{\dot{y}_t}{y_t} = (\alpha - 1) \frac{\dot{F}_t}{F_t} + \sigma \frac{\dot{A}_t}{A_t}$ with some portion of growth due to specialization or consumers' love of variety. Removing this love of variety effect by specifying $\alpha = 1$ does not remove fully-endogenous growth. With labor market clearing, per capita growth can also be written as $\frac{\dot{y}_t}{y_t} = (\theta - 1) \frac{\dot{F}_t}{F_t} + \sigma \gamma s \frac{L_t}{F_t}$. As F_t is itself a function of L_t, growth is not dependent on the size of the economy but on research effort and technological improvement at the firm or sector level. Increased research effort by firms or industries influence the growth rate such that growth is fully-endogenous without scale effects.

These models borrow techniques from Industrial Organization by recognizing that the entry of variety expanding ideas in response to population growth mitigates increasing returns to innovation by spreading research effort over a wider variety of ideas. In much the same way as free entry constrains the impact of increasing returns on prices, the entry of variety expanding ideas constrains the impact of increasing returns from a growing population on innovation. However, since the linear functional form of innovation in first generation models applies at the firm level this linear assumption essentially performs the same task as it did in first generation models: preventing explosive growth. That is, these models use the entry of new varieties to control for the impact of increasing returns from population growth in order to remove the scale effect but disallow 'excessive' or 'insufficient' increasing returns to cumulative ideas by making a strict linear assumption. This strict assumption leads to the key criticism of Schumpeterian models known as the linearity critique.

There are two so-called 'knife-edge' assumptions used to criticize Schumpeterian models. First and foremost, growth models require that at least one differential equation is effectively assumed to be linear in order to sustain growth and avoid explosive growth. That is, there is always a differential equation $\dot{X} = fX$ where X is a model specific variable that grows indefinitely and f can have a variety of functional forms but is effectively constant with respect to X. Growiec (2007) provides a proof that such a linear function is required, although Peretto (2018) shows that linearity is only required to be a property of the steady state. In neoclassical and first generation models, X refers to technology or ideas. In semi-endogenous models, X refers to population. In Schumpeterian models, X refers to product quality. The linearity critique asks that such strict linear assumptions be justified (Jones, 1999, 2005). The assumption of linear population growth is not controversial since it is well-known that population expands in proportion to its size. While semi-endogenous models are still subject to a linear assumption, Jones (1997, 2005) argue that the logical place for assuming linearity is within population growth, as in Jones (1995a), and that this is the least obtrusive approach. However it comes with the unfortunate side-effect of growth not remaining fully-endogenous.[8] For first-generation models and the Schumpeterian models by Young (1998); Dinopoulos & Thompson (1998); Peretto (1998) and Howitt (1999), the linearity critique applies to the growth equation itself. Secondly, product proliferation in Schumpeterian models is assumed exactly proportional to the growth of some resource endowment such as population. This mitigates the scale effect but introduces another critical 'knife-edge' assumption.

Li (2000) argues that Schumpeterian models have an additional assumption of no knowledge spillovers between vertical and horizontal dimensions of R&D and that by relaxing this additional assumption, these models regress to semi-endogenous growth. This reliance on several critical 'knife-edge' assumptions is argued to be the weakness of Schumpeterian models. Laincz & Peretto (2006) prove that product variety is indeed in proportion to population similar to how population growth is in proportion to its size, but the linearity critique still applies to assumptions about firm level growth. Temple (2003) argues that 'knife-edge' assumptions are merely tools to simplify the analysis into tractable models and that the power of a model should rest on its ability to reflect real world evidence. Romer (1994) is even explicit about this. In this sense, Schumpeterian models shouldn't be rejected on the basis of modeling assumptions if evidence can be found in favor of their prediction that support for R&D does indeed support long run growth. This evidence now appears overwhelming, but the dispute between restrictive assumptions and empirical evidence has continued.

3.4 *4G* endogenous growth without scale effects

Peretto (2018) resolves *both* of these criticisms with market responses that achieve linearity in the steady state and only in the steady state, thereby not relying on any assumed linearity or proportional product proliferation. By defining each generation according to the techniques used to negate explosive growth and the extent of restrictive assumptions, this contribution can be viewed as the first of a fourth generation of models in which endogenous growth without scale effects is modeled without any linear or knife-edge assumptions at all. Schumpeterian models may define innovation in ways that are not explicitly linear, but the critique still applies if linearity is effectively assumed in the growth equation. Similarly, Schumpeterian models may specify a market mechanism to define market structure such that product proliferation is effectively proportional to the endowment, but still require a linearity assumption in the growth equation itself (see for example Peretto (1998) and Laincz & Peretto (2006)). But the distinctive characteristic of 4G models is that growth remains fully-endogenous without any linear assumptions at all.

In this class of models, starting with Peretto (2018), the f described in the linearity critique above is not constant but the equilibrium outcome of market responses to X that becomes constant in the steady state and only in the steady state. That is, f is a function of X itself and other things that becomes constant and independent of X in the steady state. In Schumpeterian models, $f$is always independent of X, so is effectively assumed constant with respect to X. Peretto (2018) uses the function

$$f \equiv \frac{A^{\kappa-1}L}{N^{1-\sigma}} \tag{9}$$

to describe non-linear product proliferation (in which $X = N$) in response to both population growth and (excessive) increasing returns to quality, and to describe non-linear quality improvement (in which $X = A$), where $\kappa \geq 1$ and $\sigma < 1$ measure returns to quality and variety respectively. As a result, the growth rates of variety and quality are themselves non-linear functions of f which is also non linear in the combination of the two (since $\sigma + \kappa \neq 1$). The only requirement is a standard free entry and no arbitrage rule in the steady state such that investment in either an improvement or a new variety always yields a standard rate of return.

The model in Peretto (2018) is able to generalize all earlier generations. If entry were particularly attractive ($\sigma \geq 1$) the model degenerates to the standard Romer (1990) model with a scale effect such that growth is simply a result of increasing variety and a preference for variety. Similarly, for $\kappa = 1$ the model is the standard Schumpeterian model with its linear requirement on returns to knowledge. In this way, the Schumpeterian model can be thought of as a special case where the linear 'critique' is simply a modeling trick to avoid dealing with explosive growth from excessive increasing returns. Finally, the semi-endogenous case is also nested in the 4G model twice. Firstly, if parameters are defined such that start-up costs for new varieties of average quality are too low then the rate of return from investing in new varieties could still exceed the rate of return from investing in quality improvements in the steady state. In this corner solution, no quality improvement occurs in the steady state and new varieties are only the result of population growth as in the semi-endogenous model.[9] Secondly, if quality improvements are increasingly difficult to discover such that $\kappa < 1$, then the logic follows that researching quality improving ideas would eventually cease such that any new varieties only match existing quality and growth is simply a result of increasing variety due to population growth. The requirement of no improvements to existing varieties seems particularly strict, but the invention of entirely *new* varieties is perhaps more semantic. On the other hand, if quality improvements are increasingly easy to find ($\kappa \geq 1$), but entrants compete away the excessive profits associated with those improvements, then it follows that non-explosive endogenous growth is sustained indefinitely. The parameters that define each generation within Peretto (2018) are described by Figure 3. As a result, the linearity critique is no longer a valid weakness for models of fully-endogenous growth.

[Figure 3 here]

The technique used in 4G models is a repetition of the free entry and imperfect competition technique applied now to 'excessive' increasing returns to the scale of innovation. In much the same way as entry constrains the impact of increasing returns on prices, the entry of new ideas constrains the impact of excessive increasing returns or population on the production of ideas. In Schumpeterian models the technique was incomplete since it applied only to variety expanding ideas due to population growth and required a linear assumption to prevent excessive increasing returns on quality improving ideas. The newest generation of models goes one step further by allowing entry to constrain the impact of increasing returns on the production of variety expanding ideas due to population and the impact of excessive increasing returns in the production of quality improvements.

Recognizing that this new approach is uniquely different to earlier generations, this class of models is referred to here as the *'4G'* class of fully-endogenous growth models without scale effects. The unique characteristic of the new class of models is that some logical market mechanism achieves the required linearity in the steady state and only in the steady state without relying on the placement of an assumption about the particular form of a differential equation.

In particular, Peretto (2018) identifies that quality improving innovations generate an opportunity for product proliferation by expanding the size of the market for all suppliers. Product proliferation and changes in demand are market responses to match the effect of any non-linearity from quality improvement, taming excessive returns to achieve linearity in the steady state and only in the steady state as well as negate the scale effect. That is, the market does not allow quality improvements to sustain excessive returns, with new varieties emerging in response to quality improving innovations and in response to changes in population.

As a result, these models imply that the linearity assumptions of Schumpeterian models are merely simplifying rather than a factual description of knowledge production. In this way, the linear assumptions in Schumpeterian models are merely a technicality that limits their application in a particular way, rather than a strong criticism. As a result, this class of models offers a resolution to the long running dispute about how to correctly remove scale effects from models of endogenous growth by reducing the linearity critique to a mere technicality.

3.4.1 Other 4G mechanisms

Such mechanisms do not only have to respond to 'excessive' increasing returns. Some mechanisms could also respond to 'insufficient' increasing returns if there is diminishing returns to particular factors for innovation. For example, Aghion & Howitt (1998, ch. 12) offer two such mechanisms to counteract diminishing returns to the knowledge stock to help justify the required linearity. In their model capital and technological progress also augment R&D. In the first mechanism, induced capital deepening offsets diminishing returns to R&D and in the second, technological progress increases labor productivity for both production and R&D. Growth remains fully-endogenous because capital accumulation and the knowledge stock grow at the same rate, despite diminishing returns in each individually. While 'augmented technological change' could be thought of as a 4G mechanism, Aghion & Howitt (1998, ch. 12) still effectively assume linearity. Similarly, product proliferation is assumed constantly proportional to population size. In this way, Aghion & Howitt (1998, ch. 12) is still placed within the Schumpeterian generation but an extended model could fit within the 4G class. For example, a 4G version might imply that capital deepening and population growth also generate opportunities for product proliferation by augmenting R&D.

It should be noted that a single mechanism such as product proliferation does not have to explain the entire source of linearity in the steady state, but the combination of mechanisms should lead to linearity in the steady state and only in the steady state. In this way, a series of mechanisms, which may have various impacts at different transition points, achieves fully-endogenous growth while constraining explosive growth without the use of any 'knife-edge' linearity assumptions. Many further market mechanisms could facilitate linearity if they are a function of X itself and their combination in equilibrium becomes constant in the steady state.

Many such secondary mechanisms are already introduced in various forms. For example, while Ramondo et al. (2016) does not remove scale effects, domestic trade costs partially offset the scale effect, representing a potential mechanism to counteract the scale effect and any non-linearities. That is, excessive increasing returns to innovation are diminished by congestion and other costs that increase as a result of and in response to the increased economic activity that is associated with such non-linearities. Aghion et al. (2017) identify that Baumol's (1967) cost disease could constrain explosive growth when artificial intelligence becomes an input to the production function. Baumol (1967) observed that if there is rapid productivity growth in some sectors then the relatively slow growing sectors become increasingly important to production. As a result, sectors with high productivity growth shrink as a share of the overall economy. Nordhaus (2008) confirms that Baumol's hypothesis is consistent with the modern growth experience since 1948. While Nordhaus (2015) finds that we are a long way from the so-called 'singularity', the identification of Baumol's cost disease as a constraint on explosive growth indicates that a similar cost disease mechanism might also lead to linearity in the steady state in a 4G model. That is, excessive increasing returns in some sectors are constrained by diminishing returns in other, slow growing but essential sectors. With more than one mechanism, each one alone does not have to achieve the required linearity but their combination is sufficient. A 4G model with mechanisms to offset non-linearities from both excessive (Peretto, 2018) and insufficient increasing returns would allow a universal specification.

3.5 Four generations of differing assumptions

While many techniques have been proposed to remove the scale effect from theoretical models of endogenous growth, each model is founded on particular assumptions and these assumptions can be categorized around common themes. This section discusses the role of these assumptions and their implications to support semi-or fully-endogenous growth without scale effects.

Jones (1995a); Kortum (1997); Segerstrom (1998) counteract the scale effect by assuming diminishing returns to knowledge accumulation and retain endogenous growth by assuming linear population growth. While Jones (1995a) assumes diminishing returns in aggregate knowledge accumulation based on empirical findings in Jones (1995b), Kortum (1997) assumes diminishing returns to innovation for additional sectors and Segerstrom (1998) assumes diminishing returns for cumulative quality improvements to each variety. Intuitively, these models all assume that the simple ideas are easier to find and researchers discover these so-called 'low hanging fruit' first such that future innovations are increasingly difficult to develop. As a result, any support for additional research effort merely finds the simple ideas faster as technology advances until increasing difficulty matches the former discovery rate. Any short term boost to growth is counteracted by diminishing returns to knowledge accumulation in the steady state. While investment in innovation is endogenous in these models, growth is considered semi-endogenous because research effort has no impact on the long run growth rate that depends exclusively upon population growth.

Since growth is no longer fully-endogenous, these models almost ignore one purpose for endogenizing investment in innovation in the first place. If the endogenous investment in effort to develop non-rival ideas is the driving force behind economic growth, it is intuitive that increased research effort would impact long run growth rates. Denying this role also leads to a possible misinterpretation of the semi-endogenous conclusion in order to reject a support role for governments in the innovation system. Proponents of semi-endogenous models should be careful to highlight the major point of agreement, that non-rivalry is a source of market failure. Scale effects are still present in a 'weak' form where per capita incomes are a function of total research effort. That is, economies with greater overall research effort can be expected to have higher per capita incomes. Although research effort has little impact on long run growth rates, government support for research is still justified on the basis of these level effects on welfare.

The assumption that the simple ideas are discovered first seems intuitive, but the prediction of a scale effect in per capita incomes seems unlikely given per capita incomes in high population countries in Asia and Africa. Jones (2005) points out that institutional differences need to be accounted for, but such tests also define economies by their borders, while the exchange of ideas as an input to innovation is global. Instead these semi-endogenous models are really predicting a 'weak' scale effect for incomes at either a global level or a functional economic area level. Accounting for domestic trade frictions partially counteracts the scale effect (Ramondo et al., 2016) implying a functional area level result, but endogenous growth isn't the only theory to predict divergences in per capita incomes by scale. Alternatively, mechanisms such as trade (Krugman, 1980; Eaton & Kortum, 2001, 2002; Melitz, 2003) and spatial mechanisms (Bond-Smith et al., 2018) can also drive scale effects by functional economic area. Therefore, so-called weak scale effects could be apparent in the real world, but theoretical models could also aim to understand the mechanisms that drive such a scale effect rather than assume it within endogenous growth itself.

Schumpeterian models mitigate the empirically-refuted scale effect by assuming that population growth results in product proliferation, retaining endogenous growth by assuming that the production of innovations is linear at the firm level. Hence these models also remain subject to the linearity critique. One further limitation of some Schumpeterian models is that support for R&D only increases growth if it targets research intensity (Young, 1998) or exclusively targets quality improvements but not pure invention (Dinopoulos & Thompson, 1998). Furthermore, these initial models are unable to explain the diminishing productivity to innovation effort relationship identified by Jones (1995b). Howitt (1999) solves this particular issue by assuming that returns to innovation diminish for additional sectors, such that both population growth and support for R&D lead to product proliferation into less innovative sectors due to greater research effort, but R&D subsidies still have a positive effect on growth within each sector that is only partially offset by the associated diminishing returns of product proliferation. Aghion & Howitt (1998, ch. 12) allow for diminishing returns to knowledge accumulation, but effectively assume linearity by augmenting research effort with capital and technology. While overall research effort might imply diminishing returns to knowledge in some form, these Schumpeterian models are supported by more empirical findings that typical research employment per firm has remained relatively constant (Laincz & Peretto, 2006). Yet all of these Schumpeterian models are still faced with the linearity critique, treating linearity as a kind of modeling trick to endogenize growth.

The latest 4G class of models finally solves this linearity issue by designing mechanisms such that linearity is an equilibrium characteristic in the steady state and only in the steady state as an endogenous response to innovation itself. As a result, 4G model parameters are not confined to the 'knife-edge' assumptions that have dominated arguments against the Schumpeterian approach. Furthermore, this development confirms that assumed linearity is indeed a modeling trick of Schumpeterian models rather than a fact about the true state of the world or a test used to reject their validity. The appeal of such mechanisms is both its intuition and its use elsewhere such as the vertical linkages that lead to product proliferation and specialization in trade models (Venables, 1996; Krugman & Venables, 1996).

These assumptions are summarized in Table 1.[10]

[Table 1 here]

4 Empirical support

The validity of the each model's assumptions is critical, but each theory is also only as good as its predictions. This section briefly summarizes the conclusions drawn from empirical studies regarding support for endogenous growth theories. A more thorough survey and analysis of the estimation strategies and empirical techniques could be a survey paper of its own and is considered beyond the scope of this article.

There are of course many ways to distinguish between empirical studies. The focus of this article is the theoretical techniques, so empirical studies are discussed here around two alternative approaches.[11] One strand of research estimates the characteristics of an ideas production function. In doing so, the coefficients can be used to identify which endogenous growth model seems to best explain rates of innovation by specifically examining the estimated productivity of research inputs. Alternatively a second strand of research contrasts endogenous growth theories directly by testing their implications. For example, by examining the relationship between population and growth or the impact of R&D effort on productivity growth.

Both approaches find support for endogenous innovation and while there is some support for semi-endogenous growth, the wealth of research supports fully-endogenous growth without scale effects. The cause of such mixed results is itself a large topic, but the analysis here suggests that it is likely to be a result of data, measurement and definitional factors. The data inputs all require proxies that have various measurement issues. Innovations are difficult to define and not consistently recorded. Many innovations go unrecorded if the idea is not patentable. The impact of each innovation varies substantially. R&D measures such as expenditure or employment only capture formal research activities and informal research effort goes unrecorded. Some models aim to predict the long run growth rate, but transitional effects can take some time to diminish, if ever. Disentangling the long run from transitional growth is highly contingent on the theoretical model applied. All of these factors make estimates of an innovation productivity function vary widely.

Such a weakness of empirical growth analysis is also reflected in Paul Krugman's criticism of the so-called 'new growth theory' that "too much of it involved making assumptions about how unmeasurable things affected other unmeasurable things."[12] This is an important point in the dispute. Even with empirical support, such evidence is still weak if data are incomplete and models rely on crude assumptions. This conclusion is also supported by theoretical discussions that assert such assumptions are merely technicalities used out of necessity, rather than facts about the true state of the world (Dalgaard & Kreiner, 2003; Temple, 2003; Growiec, 2007). On this basis, this article focuses on critically analyzing the theoretical techniques to remove scale effects, but good theories are still not developed in isolation of empirical evidence.

4.1 Ideas production function

Ideas production functions have been estimated even prior to the development of semi-endogenous or Schumpeterian models. In fact, Nordhaus (1969) estimates an early ideas production function that was not supported by the data. Later studies find a more reliable specification and can be used to indicate which model best explains endogenous growth by examining the productivity of the inputs to innovation. Higher quality studies estimate the function over long periods and include estimates of international knowledge spillovers, reflecting the internationally non-rival nature of knowledge.

More recent studies that directly analyze the functional form of endogenous growth theories are easier to interpret. With a functional form of $\dot{A}_t = \gamma L_{At}^{\lambda} A_t^{\phi}$, the linearity critique that $\phi = 1$ can be empirically tested. Overall, the evidence is mixed with some studies finding that $\phi < 1$ and others finding that the hypothesis $\phi = 1$ cannot be rejected.

It is difficult to measure the inputs to the ideas production function directly, so all empirical studies rely on proxies and approximations. The proxy for knowledge spillovers is typically patents or TFP while the proxy for effort is R&D spending or R&D workers. In particular, Griliches (1990) argues that patents are probably the best measure of both innovation output and the knowledge stock.

Jaffe (1986) is probably the first to study the role of knowledge spillovers in R&D using patent data finding that the productivity of spillovers was less than one, so apparently supporting semi-endogenous theories. However, it is difficult to interpret these early knowledge production functions in terms of an endogenous growth theory that did not yet exist.

Porter & Stern (2000) estimate the aggregate ideas production function directly from endogenous growth models using non-resident patent data finding that ideas production is declining in the world-wide stock of ideas, but the hypothesis that $\phi = 1$ cannot be rejected. The use of international patents is important for endogenous growth since knowledge is non-rival, so knowledge spillovers are not necessarily defined by borders. In absence of international data, some studies use other proxies to represent international knowledge spillovers such as particular classes of imports. An alternative approach in Furman et al. (2002) considers a broader question of national innovation capacity across a sample of 17 OECD countries. When using GDP or GDP per capita to represent the knowledge stock, Furman et al. (2002) find that $\phi = 1$ cannot be rejected, but a specification using patents finds that $\phi < 1$. Abdih & Joutz (2006) also use two specifications: one using patents and one that follows Jones (1995b) specification with TFP to represent the knowledge stock. The first specification finds $\phi = 1$ in favor of fully-endogenous growth while the second specification finds that $\phi > 1$. In particular, they suggest that the diminishing relationship found by Jones (1995b) really represents a weak relationship between the knowledge stock and TFP growth, rather than a characteristic of knowledge production. That is, their results imply that ideas can be explosive even if TFP growth is not.

Recent studies are more conclusive in their respective results, but there is no complete consensus in favor of one generation or another. Ang & Madsen (2011) produce a range of estimates that typically find $\phi = 1$ whether research effort is measured by expenditure or workers. Barcenilla-Visús et al. (2014) find support for semi-endogenous growth but emphasizes the role of R&D support for its transitional effects. Furthermore, they find partial support for Schumpeterian models to play a role in technology transfer. That is, for economies behind the technology frontier, support for R&D assists with catching up, increasing growth rates as technology tends towards the frontier. Countries on the frontier however, are unable to use support for R&D to boost long run growth rates. Ang & Madsen (2015) use a long historical data set of patent applications in the US, UK, Germany and Asian Tiger economies from 1870 to 2010 typically finding that $\phi \approx 1$ with very low standard errors. While the exact estimates are often just below one, other coefficients are inconsistent with the semi-endogenous or first generation models, such that findings overall support the Schumpeterian model. In particular, they find evidence of product proliferation effects and that expansions of R&D due to population growth were neutralized by increases in product variety, supporting the Schumpeterian model.

Given the wealth of research, a meta-analysis of empirical studies could be expected to yield insights. Recently, Neves & Sequeira (2018) summarize the empirical evidence in a meta-analysis on the productivity of knowledge spillovers and find in favor of semi-endogenous theories over Schumpeterian growth models, yet their estimate of ϕ is still very close to one. Neves & Sequeira (2018) also note that their findings would be overturned if much innovation cannot be recorded as patents. The conclusion suggests that the difficulty to measure innovation or knowledge could drive support for semi-endogenous growth, rather than imply facts about the true state of the world. Upon closer inspection of the studies included in the meta-analysis, the conclusion may be premature. For example, Ang & Madsen (2015) find estimates of ϕ that are just below one but still conclude in favour of Schumpeterian growth. A broader treatment of the empirical literature as summarized here uses a range of other measures to reach a more definitive conclusion and overwhelmingly finds in favor of fully-endogenous growth.

A summary of the main conclusions of these empirical studies is shown in the first two columns of Table 2.[13] The conclusions from several more papers are included in the table, but this is not an exhaustive list. The conclusions are sensitive to the choice of variable for the knowledge stock. If patents or GDP per capita represent the knowledge stock, evidence is mixed, but other proxies for the knowledge stock such as TFP cannot reject the fully-endogenous model.

4.2 Direct comparison

Given the inherent difficulty measuring knowledge and innovation, a more suitable approach that focuses on the characteristics of productivity growth could be more useful. These studies consider the expected stationarity of particular variables and the relationships between variables such as population and growth.

As in studies of the knowledge production function, studies prior to the development of formal endogenous growth theories can also indicate support for particular endogenous growth models. Early studies note that there is very little evidence of declining rates of return to R&D (Griliches, 1986, 1990; Sveikauskas, 1990), in contrast to the later predictions of the semi-endogenous models. Adams & Jaffe (1996) find that firm level R&D characteristics are the key determinants of productivity growth, diluting aggregate industry returns, and therefore supporting later Schumpeterian models.

Recent studies that directly compare the theories also tend to support fully-endogenous models. Ha & Howitt (2007), Madsen (2008) and Ang & Madsen (2011) examine long-run trends of R&D and TFP finding in favor of Schumpeterian growth models. Ha & Howitt (2007) find that the semi-endogenous model is not supported by the evidence, because TFP growth has remained constant while the growth rate of R&D has been decreasing. Instead the Schumpeterian model provides a better explanation for long run growth trends. Examining the growth equation, Madsen (2008) finds limited support for the semi-endogenous approach and strong support for the Schumpeterian model. Similar to results for the ideas production function, estimates are more significant when innovative activity is represented by R&D measures than patents. Ang & Madsen (2011) find evidence in support for either semi-endogenous or Schumpeterian growth when R&D is measured by expenditure, but only for the Schumpeterian model when R&D is measured by workers. Examining an integrated model also finds support for the Schumpeterian model. Ha & Howitt (2007), Madsen (2008) and Ang & Madsen (2011) also use co-integration tests for the stationarity of particular variables to compare the two theories, all finding substantial evidence in favor of Schumpeterian models. Sedgley & Elmslie (2010) also use co-integration tests of productivity trends and include transitional dynamics finding in favor of fully-endogenous theories.

Yet evidence continues to also be found in favor of the semi-endogenous approach. Jones (2002) argues that transitional factors such as continually increasing educational attainment and rising research intensity explain as much as 80 per cent of US growth, with the remaining 20 per cent a result of population growth through a semi-endogenous idea production function. Most recently, Bloom et al. (2018) show that ideas are becoming progressively more difficult to discover, supporting the assumptions used in Semi-endogenous models. Yet such a discovery also supports the assumptions used in Howitt (1999), which retains fully-endogenous growth with increasing R&D effort and population growth by assuming that innovation productivity diminishes for additional varieties.

A summary of the papers discussed here and others is shown in the last two columns of Table 2. The conclusions from several more papers are included in the table, but this is not an exhaustive list. The conclusions are also sensitive to the choice of approach. Studies examining the trend of innovation productivity can find mixed results, but studies that examine the overall trend of multiple variables using co-integration tests find in favor of Schumpeterian models.

4.3 Summary

The table below summarizes the alternative approaches of a sample of articles and whether the findings support either semi-endogenous or Schumpeterian models, but this is not an exhaustive list. Studies can be listed more than once, if the evidence is mixed.

The evidence in favor of fully-endogenous growth is now overwhelming. Mixed results seem to be largely caused by empirical and measurement techniques. Continued debate and counter-evidence is largely fueled by theoretical issues. Hence the review and analysis of the theoretical techniques surveyed in this article is an important step to resolving the dispute.

[Table 2 here]

4.4 4G models

Schumpeterian models can be considered a special case of the 4G class, such that evidence in favor of fully-endogenous Schumpeterian growth also supports 4G models. The distinguishing characteristic of 4G models is that linearity is a characteristic only of the steady state, as a result of some market response mechanism. Instead of attempting to validate or criticize Schumpeterian over semi-endogenous models, empirical research can now turn to estimating the impact of the various market mechanisms that achieve linearity in the steady state. While this latest generation of endogenous growth models without scale effects is perhaps too young to already be the subject of rigorous empirical scrutiny, some studies may indirectly imply specific components that could be included as mechanisms that form this new class of fully-endogenous growth models without scale effects.

In particular, Laincz & Peretto (2006) provide support for the product proliferation mechanism later used in Peretto (2018). In this paper Laincz & Peretto (2006) suggest that aggregate measures of R&D employment misrepresent the empirical predictions of Schumpeterian models. Instead, they find that R&D effort per firm has not increased over time so and does indeed spread out across more firms due to product proliferation.

Product proliferation does not have to be the only mechanism that achieves linearity. For example, Ramondo et al. (2016) show how trade frictions partially tame the predicted scale effect. More specific mechanisms such as transport prices, port capacities and congestion costs also likely contribute to mitigating scale effects at some functional economic area level. This highlights the key achievement of this fourth generation of models, in that research can now focus on both the mechanisms that achieve linearity and the mechanisms that lead to any observed scale effect at the economic area level such as agglomeration economies (Bond-Smith et al., 2018).

4.5 Other factors

In recent periods, productivity growth appears to have slowed, despite increases in technology. The so-called 'productivity paradox' is often characterized by quoting Robert Solow's remark that "You can see the computer age everywhere but in the productivity statistics."[14] While the productivity paradox is sometimes seen as evidence for semi-endogenous growth theories, many studies also indirectly dispute the suggestion that ideas are becoming harder to find by emphasizing other factors that contribute to the recent growth experience of many countries. For example, several studies focus on diminishing returns to capital accumulation (De Long & Summers, 1991; Mankiw et al., 1992; Young, 1995) which applies in all its forms including human capital. Alternatively, a recent series of papers implies that ideas have always been difficult to find and that the growth experience since the industrial revolution is due to a series of one off events, which might now be coming to an end (Gordon, 2016; Madsen, 2018). For example education rates and female labor force participation eventually reach saturation, preventing increases in these components from contributing further to economic growth rates.

5 A dispute resolved?

After four generations of endogenous growth theory, this long running dispute may now be finally resolved. The key critique that has stimulated the dispute is resigned to a mere technicality by the latest 4G class of models with little limitation on model parameters (Peretto, 2018). This section discusses the implications if the scale effects dispute is now resolved.

5.1 Endogenous growth theories

Peretto's (2018) resolution to the scale effects dispute allows a reinterpretation of Schumpeterian modeling techniques. In particular, the assertion that the so-called 'knife-edge' assumption of linearity is a modeling necessity rather than an empirical requirement (Dalgaard & Kreiner, 2003; Temple, 2003; Growiec, 2007) is confirmed. Linear assumptions are only a simplifying tool in Schumpeterian models, rather than a statement of fact about the true state of the world that can be used as a test of their validity. Linear assumptions are not required for fully-endogenous growth without scale effects if linearity is achieved as an equilibrium outcome in the steady state and only in the steady state. Therefore, the key critique that Schumpeterian models rely on knife-edge assumptions is rendered meaningless, as it only facilitates how the results of fully-endogenous growth models should be interpreted, rather than their validity. On this basis, critiques of Schumpeterian models fail on two counts. Firstly, the linearity critique applies only to specific simplifying assumptions rather than the real world nature of fully-endogenous growth that is implied by Schumpeterian models. And secondly, the weight of empirical evidence supports the positive impact of support for research effort, at least over any reasonable, measurable period.

In 4G models, acceptable parameter values for fully-endogenous growth without scale effects are no longer restricted to a 'knife-edge' but a 'wide highway' of possible calibrations (Peretto, 2018). On this basis, Peretto (2018) offers a substantial development beyond Schumpeterian models. Both semi-endogenous and 4G models rely on alternative sides of a 'cliff edge' assumption about whether ϕ is greater than or less than one. But the development of 4G models is possibly broader than that. If market mechanisms in new 4G models can be established that also respond to innovations which are less than linear in order to achieve linearity in the steady state, the 'cliff edge' of Schumpeterian models becomes a horizon with no upper or lower bound on assumed parameter values to achieve fully-endogenous growth without scale effects.

5.2 A maturing of modeling techniques

There are many ways in which each generation can be identified as distinctly different from earlier models, but the synthesis here identifies the development of modeling techniques along three conceptual themes.

Firstly, first generation, Schumpeterian and 4G models can be thought of as repeating the standard modeling techniques to constrain the impact of increasing returns, but each generation applies it in a further dimension. Increasing returns lead to substantial changes in how economists think about, competition, equilibrium, prices and output throughout economics. This had a major impact on international trade (Krugman, 1979), development (Krugman, 1992), economic growth (Romer, 1990) and economic geography (Krugman, 1991). The analysis here suggests that the application of increasing returns to the concept of ideas has followed a similar developmental pattern that has now reached maturity. This application goes beyond equilibrium prices to understand its impact on the production of ideas, because ideas are a non-rival input to *and* output from innovation. Adding invention to the standard neoclassical model, Nordhaus (1969) identifies that free entry for inventors means the cost of invention uses the entire monopoly profit from successful invention. For first generation models, free entry and imperfect competition allows increasing returns at the firm level and constrains its impact on equilibrium prices, avoiding the need to assume perfect competition. But linear assumptions about the nature of growth and restrictive assumptions about population are required to avoid explosive growth. In Schumpeterian models, the entry of variety-expanding ideas in response to population growth constrains the impact of increasing returns on the aggregate scale of research effort by spreading it more thinly, avoiding the need to assume linear population growth and retaining fully-endogenous growth. But the model requires an assumption of constant (linear) returns to the scale of the knowledge stock in each variety to find quality improving innovations in order to avoid explosive growth. Peretto (2018) takes the entry technique one dimension further such that the entry of new varieties in response to excessive returns enables increasing returns to both the scale of population and the scale of cumulative quality improvements while constraining the impact of increasing returns on steady state growth without the need for any linear assumptions at all.

Secondly, semi-endogneous and 4G models differ from first generation and Schumpeterian models because constant or linear returns to the scale of production of new ideas is a characteristic of the balanced growth path or steady state rather than a result of particular linear assumptions. This is particularly appealing because linearity is a characteristic of equilibrium that emerges endogenously.

Thirdly, the different approaches vary by the extent that restrictive or simplifying assumptions remain to negate the impact of the scale effect if the particular modeling technique is insufficient. First generation models must assume that there is no population growth and assume linear returns to the cumulative scale of knowledge to avoid explosive growth. Semi-endogenous models assume linear population growth and diminishing returns to the cumulative scale of knowledge. In a semi-endogenous model with horizontal and vertical innovation dimensions, product proliferation must be assumed to be more than linear to population growth to avoid explosive growth if returns to knowledge are greater than linear. Schumpeterian models assume linear returns to the cumulative scale of knowledge at the sector level and require product proliferation to be proportional to population growth. Given the absence of any linear or diminishing returns assumptions at all in Peretto (2018), the dispute over scale effects in the theory of endogenous growth may be finally reaching a conclusion.

This synthesis also implies that the various generations of theories should be thought of as a standard maturing of modeling techniques. In much the same way as earlier neoclassical growth models used technological change to mitigate diminishing returns to capital in order to sustain growth, the assumption of diminishing returns to the scale of cumulative knowledge in semi-endogenous models mitigates increasing returns to the scale of research effort. Similarly, assumptions of perfect competition allowed equilibrium prices to be found in neoclassical models in the absence of suitable techniques to model imperfect competition in much the same way as assumed linear quality improvements allow endogenous growth in Schumpeterian models in the absence of Peretto's (2018) product proliferation mechanism. The neoclassical growth model remains a powerful tool for examining growth despite these simplifying assumptions and an improved understanding of technological change.

Viewed in this way, it seems that knife-edge assumptions are simply modeling tricks that are also not necessarily critical to each models' findings, but may simply limit its application. Paul Romer is even explicit about this. In describing the historical use of assumptions such as linearity, Romer (1994) describes that Romer (1986) was robust to a variety of specifications where $\phi > 1$, in much the same way as Peretto (2018), but that it required additional and unnecessarily sophisticated mathematical analysis that studied the phase plane of the nonlinear differential equation system rather than solving a simple linear differential equation. On this basis, Romer (1990) pursued elegance with a simple special (linear) case. In this way, Peretto (2018) can also be thought of as endogenous growth theory now completing a full circle, but the addition of ideas in two dimensions reduces the mathematical sophistication issue to the standard optimization problem.

5.3 Recent empirical studies

Recent empirical studies may help to understand the 4G mechanisms that achieve linearity in the steady state. Acemoglu & Restrepo (2017, 2018) contend that population decline may be accelerating productivity growth by stimulating the adoption of automation technologies. This implies the invention and expansion of new sectors in response to non-linearities in population growth. It is not so surprising that such processes would also apply to other non-linearities that affect productivity growth. In this case, population decline increases the rate at which established industries exit the market to be replaced by new technologies. In this way, industry churn could represent a further 4G mechanism. This implies the importance of searching for evidence of the market responding to all non-linearities that affect growth. These will provide clues to the market mechanisms, such as product proliferation, that achieve linearity in the steady state.

In a similar manner, the findings in Bloom et al. (2018) that ideas are becoming harder to find is perhaps evidence of a process of decline for established industries but not new industries. The focus on individual sectors may mask product proliferation from new sectors that are not measured by Bloom et al. (2018).

These empirical studies offer insights into the types of mechanisms that could both cause and correct for non-linearities for innovation. On this basis, a reinterpretation of other empirical results may lend support for the product proliferation mechanism in Peretto (2018), the industry churn mechanism proposed here or indicate other mechanisms. In light of this, further re-examinations of studies into the relationships between market structure, industry characteristics and technological change offers researchers an avenue to identify further 4G mechanisms that endogenously achieve linearity in the steady state.

6 Concluding remarks

Four generations of endogenous growth theory represent a cumulative process of new economic knowledge in much the same way as Romer's (1990) model predicted. Each generation incrementally builds upon earlier ideas. In particular, this article shows that expanding ideas about growth can be interpreted as a repackaging of old ideas in new ways. This repackaging shows how knowledge is explosive, even if TFP growth is not. The important factor is understanding how ideas translate into productivity growth.

The dispute over how to handle explosiveness is a particularly interesting dimension of this now broad field of research. The analysis here suggests that growth theory has followed a repeated developmental pattern that now applies solutions to solve for equilibrium under increasing returns to the concept of idea production. By synthesizing these developments into four generations of endogenous growth models this article has shown that growth theorists may be reaching a concluding chapter in this long running debate.

Finally resolving such a long running dispute requires several factors to align. While product proliferation has a strong micro-foundation, such 4G mechanisms require empirical scrutiny to confirm their role in aggregate, both through new studies and through a re-interpretation of existing studies. Agreement between researchers also requires avoiding the political motives that may have lead to misinterpretations of theoretical growth models (Romer, 2015). Indeed the advantage of this latest generation of models is that researchers no longer need to be distracted by simplifying technical assumptions.

At the same time, ideas lead to new ideas. Researchers should not settle for individual mechanisms to achieve linearity in the steady state as there are likely to be other components. This leads the so-called new growth theory in an all new direction. Researchers can now focus on both searching for and understanding the market responses to innovation which achieve linearity as a steady state outcome that mitigates excessive or insufficient returns to innovation, rather than arguing over modeling technicalities. This also allows the addition of micro-detail to understanding innovation and endogenous market structures as well as the policy implications of such detail. In doing so, new perspectives of economic growth phenomena such as the impact of population aging(Acemoglu & Restrepo, 2017, 2018) will become apparent because the focus of research can shift away from discussing modeling technicalities.

In an early survey regarding the development of Endogenous Growth (Romer, 1994), Paul Romer states that "It is likely that this pattern [of theory development] will repeat itself..." It now seems that a similar pattern did indeed repeat. Increasing returns in the ideas production function has gone from being constrained by linearity, to constrained by diminishing returns to knowledge, to expanding ideas and constrained by linearity at the industry level, to an explicit model of expanding ideas in all directions. In many ways the concept of explosive ideas has been well recognized. The confusion lies in the relationship between the non-rivalry of ideas and its transmission to TFP growth. On this there is perhaps still a long way to go. Techniques remain poorly developed for endogenous growth models to include the multifarious and often immeasurable component embodied by ideas. Other perspectives and measures of knowledge, such as relatedness (Boschma, 2005; Frenken & Boschma, 2007; Neffke et al., 2011; Figueroa et al., 2018), complexity (Hidalgo & Hausmann, 2009; Balland & Rigby, 2017), nestedness (Bustos et al., 2012), diversification (Neffke et al., 2018), specialization (Feldman & Audretsch, 1999), diffusion (Keller, 2004) and spillovers (Audretsch & Feldman, 1996) as well as their combinations (Balland et al., 2018) are also required to incorporate the concept of non-rival ideas into standardized growth theory while avoiding or negating modeling issues such as scale effects. With such a research agenda, Charles Jones's comment that "it may be difficult to compensate [Paul Romer] adequately for the knowledge spillovers"[15] seems a very fitting description of the continued effort to build upon Paul Romer's idea about new ideas.

Notes

[1]Constant growth in the semi-endogenous model is a result of a linear assumption regarding population growth, but this function is not controversial since it is generally accepted that population grows in proportion to its size.

[2]William Nordhaus (2018) Nobel Prize in Economics Press Conference, Yale University, Streamed live on 8 October, 2018, https://youtu.be/zQw-TJxfl-Q, accessed 6 March 2019.

[3]Kremer (1993) shows that there is perhaps such a scale effect in the very long run. Galor & Weil (2000) describe three regimes over the course of history in which the scale effect characterizes Malthusian and Post-Malthusian regimes but not modern economic growth.

[4]See for example, Zachariadis (2003); Griffith et al. (2004); Laincz & Peretto (2006); Ha & Howitt (2007); Ulku (2007); Madsen (2008, 2010); Madsen et al. (2010); Ang & Madsen (2011); Sedgley & Elmslie (2010); Greasley et al. (2013); Bollard et al. (2016). This is not an exhaustive list.

[5]See for example, Jones (1999, 2002); Jones (2009, 2010); Barcenilla-Visús et al. (2014); Bloom et al. (2018). This is not an exhaustive list

[6]Paul Romer (2016) Economic Growth, https://paulromer.net/economic-growth/, accessed 6 March 2019.

[7]See Jones (1999) and Dinopoulos & Thompson (1999) for earlier comparisons of 'first generation' endogenous growth models, 'semi-endogenous' growth models and 'Schumpeterian' endogenous growth models without scale effects.

[8]Jones (1997) is technically a fully-endogenous model without scale effects through endogenous fertility, but the innovation function is the same as the standard semi-endogenous model. As the engine of long-run growth is only endogenous fertility it is grouped with semi-endogenous models.

[9]The semi-endogenous case results if either of two inequalities fails in Peretto (2018, p. 56). These two requirements for endogenous growth can be rearranged as specific requirements on the start-up cost parameter of all new varieties. The intuition for the effect of both of these inequalities on equilibrium is the same: that variety-expanding ideas have greater returns than quality improving ideas.

[10]These papers of course show substantially more nuance and insight than discussed here. Nor is this an exhaustive list of papers that develop endogenous growth models without scale effects. The table and discussion are intended to focus on the key assumptions used in seminal papers to remove the scale effect.

[11]Thank you to an anonymous referee for this suggested framework.

[12]See Krugman, P. (2013) 'The New Growth Fizzle', The Conscience of a Liberal, The New York Times, August 18, 2013. Available at https://krugman.blogs.nytimes.com/2013/08/18/the-new-growth-fizzle/, accessed 7 September 2018.

[13]The meta-analysis in Neves & Sequeira (2018) is not included in the table, because it includes regressions with both patents and non-patent indicators as the knowledge stock.

[14]Solow, Robert M. (1987) 'We'd better watch out', New York Times (July 12), Book Review, 36.

[15]Jones, C. (2018) 'New ideas about new ideas: Paul Romer, Nobel laureate', Vox, available at https://voxeu.org/article/new-ideas-about-new-ideas-paul-romer-nobel-laureate, acccessed 15 October 2018.

References

Abdih, Y. & Joutz, F. (2006). Relating the Knowledge Production Function to Total Factor Productivity: An Endogenous Growth Puzzle. *IMF Staff Papers*, 53(2), 1–3.

Acemoglu, D. & Restrepo, P. (2017). *Secular stagnation? The effect of aging on economic growth in the age of automation*. Working Paper 23077, National Bureau of Economic Research.

Acemoglu, D. & Restrepo, P. (2018). *Demographics and automation*. Working Paper 24421, National Bureau of Economic Research.

Adams, J. & Jaffe, A. (1996). Bounding the effects of r&d: An investigation using matched establishment-firm data. *RAND Journal of Economics*, 27(4), 700–721.

Aghion, P. & Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(2), 323–51.

Aghion, P. & Howitt, P. (1998). *Endogenous Growth Theory*, volume 1 of *MIT Press Books*. The MIT Press.

Aghion, P., Jones, B. F., & Jones, C. I. (2017). *Artificial Intelligence and Economic Growth*. NBER Working Papers 23928, National Bureau of Economic Research, Inc.

Ang, J. & Madsen, J. (2011). Can second-generation endogenous growth models explain productivity trends and knowledge production in the asian miracle economies. *Review of Economics and Statistics*, 93(4), 1360–1373.

Ang, J. B. & Madsen, J. B. (2015). What drives ideas production across the world? *Macroeconomic Dynamics*, 19(1), 79–115.

Arrow, K. (1962). The economic implications of learning by doing. *The Review of Economic Studies*, 29, 155–173.

Audretsch, D. B. & Feldman, M. P. (1996). R&D Spillovers and the geography of innovation and production. *American Economic Review*, 86(3), 630–40.

Balland, P.-A., Boschma, R., Crespo, J., & Rigby, D. L. (2018). Smart specialization policy in the european union: relatedness, knowledge complexity and regional diversification. *Regional Studies*, 0(0), 1–17.

Balland, P.-A. & Rigby, D. (2017). The geography of complex knowledge. *Economic Geography*, 93(1), 1–23.

Barcenilla-Visús, S., López-Pueyo, C., & Sanaú-Villarroya, J. (2014). Semi-endogenous versus fully endogenous growth theory: A sectoral approach. *Journal of Applied Economics*, 17, 1–30.

Baumol, W. J. (1967). Macroeconomics of unbalanced growth: The anatomy of urban crisis. *The American Economic Review*, 57(3), 415–426.

Bloom, N., Jones, C. I., Van Reenen, J., & Webb, M. (2018). Are ideas getting harder to find? *Working paper.*

Bollard, A., Klenow, P., & Li, H. (2016). Entry costs rise with development. *Unpublished manuscript, Stanford University.*

Bond-Smith, S., McCann, P., & Oxley, L. (2018). A regional model of endogenous growth without scale assumptions. *Spatial Economic Analysis*, 13(1), 5–35.

Boschma, R. (2005). Proximity and innovation: A critical assessment. *Regional Studies*, 39(1), 61–74.

Bustos, S., Gomez, C., Hausmann, R., & Hidalgo, C. A. (2012). The dynamics of nestedness predicts the evolution of industrial ecosystems. *PLoS ONE*, 7(11), 1–8.

Cozzi, G. (2017a). Combining semi-endogenous and fully endogenous growth: A generalization. *Economics Letters*, 155, 89 – 91.

Cozzi, G. (2017b). Endogenous growth, semi-endogenous growth... or both? a simple hybrid model. *Economics Letters*, 154, 28–30.

Dalgaard, C.-J. & Kreiner, C. T. (2003). Endogenous growth: A knife edge or the razor's edge? *The Scandinavian Journal of Economics*, 105(1), 73–85.

De Long, J. B. & Summers, L. (1991). Equipment investment and economic growth. *The Quarterly Journal of Economics*, 106(2), 445–502.

Dinopoulos, E. & Thompson, P. (1998). Schumpeterian Growth without Scale Effects. *Journal of Economic Growth*, 3(4), 313–35.

Dinopoulos, E. & Thompson, P. (1999). Scale effects in schumpeterian models of economic growth. *Journal of Evolutionary Economics*, 9(2), 157–185.

Dixit, A. K. & Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3), 297–308.

Eaton, J. & Kortum, S. (2001). Technology, trade, and growth: A unified framework. *European Economic Review*, 45(4), 742 – 755. 15th Annual Congress of the European Economic Association.

Eaton, J. & Kortum, S. (2002). Technology, Geography, and Trade. *Econometrica*, 70(5), 1741–1779.

Feldman, M. P. & Audretsch, D. B. (1999). Innovation in cities: Science-based diversity, specialization and localized competition. *European Economic Review*, 43, 409–429.

Figueroa, C. J., Jun, B., Glaeser, E. L., & Hidalgo, C. (2018). *The Role of Industry, Occupation, and Location-Specific Knowledge in the Survival of New Firms*. Working Paper 24868, National Bureau of Economic Research.

Frenken, K. & Boschma, R. A. (2007). A theoretical framework for evolutionary economic geography: industrial dynamics and urban growth as a branching process. *Journal of Economic Geography*, 7(5), 635–649.

Furman, J. L. & Hayes, R. (2004). Catching up or standing still?: National innovative productivity among 'follower' countries, 1978-1999. *Research Policy*, 33(9), 1329 – 1354. What do we know Innovation? Selected papers from an International Conference in honour of Keith Pavitt.

Furman, J. L., Porter, M. E., & Stern, S. (2002). The determinants of national innovative capacity. *Research Policy*, 31(6), 899 – 933.

Galor, O. & Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *The American Economic Review*, 90(4), 806–828.

Gordon, R. J. (2016). Perspectives on the rise and fall of american growth. *American Economic Review*, 106(5), 72–76.

Gordon, R. J. (2018). *Why Has Economic Growth Slowed When Innovation Appears to be Accelerating?* Working Paper 24554, National Bureau of Economic Research.

Greasley, D., Madsen, J. B., & Wohar, M. E. (2013). Long-run growth empirics and new challenges for unified theory. *Applied Economics*, 45(28), 3973–3987.

Griffith, R., Redding, S., & Reenen, J. V. (2004). Mapping the two faces of R&D: Productivity growth in a panel of OECD industries. *The Review of Economics and Statistics*, 86(4), 883–895.

Griliches, Z. (1986). Productivity, R&D, and the Basic Research at the Firm Level in the 1970's. *American Economic Review*, 76(1), 141–154.

Griliches, Z. (1990). Patent statistics as economic indicators: A survey. *Journal of Economic Literature*, 28(4), 1661–1707. cited By 36.

Grossman, G. M. & Helpman, E. (1991). Quality ladders in the theory of growth. *Review of Economic Studies*, 58(1), 43–61.

Growiec, J. (2007). Beyond the linearity critique: the knife-edge assumption of steady-state growth. *Economic Theory*, 31(3), 489–499.

Ha, J. & Howitt, P. (2007). Accounting for trends in productivity and R&D: A Schumpeterian critique of cemi-endogenous growth theory. *Journal of Money, Credit and Banking*, 39(4), 733–774.

Helpman, E. (1981). International trade in the presence of product differentiation, economies of scale and monopolistic competition : A Chamberlin-Heckscher-Ohlin approach. *Journal of International Economics*, 11(3), 305–340.

Helpman, E. (1992). Endogenous macroeconomic growth theory. *European Economic Review*, 36(2-3), 237–267.

Helpman, E. & Krugman, P. (1985). *Market structure and foreign trade: Increasing returns, imperfect competition, and the international economy*. MIT press.

Hidalgo, C. A. & Hausmann, R. (2009). The building blocks of economic complexity. *Proceedings of the National Academy of Sciences*, 106(26), 10570–10575.

Howitt, P. (1999). Steady endogenous growth with population and r & d inputs growing. *Journal of Political Economy*, 107(4), 715–730.

Hu, M.-C. & Mathews, J. A. (2005). National innovative capacity in east asia. *Research Policy*, 34(9), 1322 – 1349.

Hu, M.-C. & Mathews, J. A. (2008). China's national innovative capacity. *Research Policy*, 37(9), 1465 – 1479.

Jaffe, A. B. (1986). Technological Opportunity and Spillovers of R & D: Evidence from Firms' Patents, Profits, and Market Value. *The American Economic Review*, 76(5), 984–1001.

Jones, B. F. (2009). The burden of knowledge and the 'death of the Renaissance Man': Is innovation getting harder? *Review of Economic Studies*, 76(1), 283–317.

Jones, B. F. (2010). Age and great invention. *The Review of Economics and Statistics*, 92(1), 1–14.

Jones, C. (2005). Growth and ideas. In P. Aghion & S. Durlauf (Eds.), *Handbook of Economic Growth*, volume 1, Part B chapter 16, (pp. 1063–1111). Elsevier, 1 edition.

Jones, C. I. (1995a). R&d-based models of economic growth. *Journal of Political Economy*, 103(4), 759–84.

Jones, C. I. (1995b). Time Series Tests of Endogenous Growth Models. *The Quarterly Journal of Economics*, 110(2), 495–525.

Jones, C. I. (1997). *Population and Ideas: A Theory of Endogenous Growth*. NBER Working Papers 6285, National Bureau of Economic Research, Inc.

Jones, C. I. (1999). Growth: With or Without Scale Effects? *American Economic Review*, 89(2), 139–144.

Jones, C. I. (2002). Sources of U.S. Economic Growth in a World of Ideas. *American Economic Review*, 92(1), 220–239.

Jones, C. I. & Romer, P. M. (2010). The new kaldor facts: Ideas, institutions, population, and human capital. *American Economic Journal: Macroeconomics*, 2(1), 224–45.

Jones, C. I. & Williams, J. C. (1998). Measuring the Social Return to R&D. *The Quarterly Journal of Economics*, 113(4), 1119–1135.

Keller, W. (2004). International technology diffusion. *Journal of Economic Literature*, 42(3), 752–782.

Kortum, S. S. (1997). Research, Patenting, and Technological Change. *Econometrica*, 65(6), 1389–1420.

Kremer, M. (1993). Population growth and technological change: One million B.C. to 1990. *The Quarterly Journal of Economics*, 108(3), 681–716.

Krugman, P. (1979). Increasing returns, monopolistic competition, and international trade. *Journal of International Economics*, 9(4), 469–479.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *The American Economic Review*, 70(5), 950–959.

Krugman, P. (1991). Increasing returns and economic geography. *Journal of Political Economy*, 99(3), 483–99.

Krugman, P. (1992). Toward a counter- counter-revolution in development theory. *The World Bank Economic Review*, 6(supplement 1), 15–38.

Krugman, P. & Venables, A. J. (1996). Integration, specialization, and adjustment. *European Economic Review*, 40(3-5), 959–967.

Laincz, C. & Peretto, P. (2006). Scale effects in endogenous growth theory: an error of aggregation not specification. *Journal of Economic Growth*, 11(3), 263–288.

Li, C.-W. (2000). Endogenous vs. semi-endogenous growth in a two-R&D-sector model. *The Economic Journal*, 110(462), 109–122.

Luintel, K. B. & Khan, M. (2009). Heterogeneous ideas production and endogenous growth: an empirical investigation. *Canadian Journal of Economics*, 42(3), 1176–1205.

Luintel, K. B. & Khan, M. (2017). Ideas production and international knowledge spillovers: Digging deeper into emerging countries. *Research Policy*, 46(10), 1738 – 1754.

Madsen, J. (2008). Semi-endogenous versus Schumpeterian growth models: testing the knowledge production function using international data. *Journal of Economic Growth*, 13(1), 1–26.

Madsen, J. (2018). Back to the past: Is growth reverting to pre-industrial levels? *Unpublished manuscript.*

Madsen, J., Ang, J., & Banerjee, R. (2010). Four centuries of British economic growth: the roles of technology and population. *Journal of Economic Growth*, 15(4), 263–290.

Madsen, J. B. (2010). The anatomy of growth in the OECD since 1870. *Journal of Monetary Economics*, 57(6), 753 – 767.

Madsen, J. B. (2016). Health, human capital formation and knowledge production: Two centuries of international evidence. *Macroeconomic Dynamics*, 20(4), 909–953.

Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth*. *The Quarterly Journal of Economics*, 107(2), 407–437.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695–1725.

Neffke, F., Hartog, M., Boschma, R., & Henning, M. (2018). Agents of structural change: The role of firms and entrepreneurs in regional diversification. *Economic Geography*, 94(1), 23–48.

Neffke, F., Henning, M., & Boschma, R. (2011). How do regions diversify over time? industry relatedness and the development of new growth paths in regions. *Economic Geography*, 87(3), 237–265.

Neves, P. C. & Sequeira, T. N. (2018). Spillovers in the production of knowledge: A meta-regression analysis. *Research Policy*, 47(4), 750 – 767.

Nordhaus, W. D. (1969). An economic theory of technological change. *The American Economic Review*, 59(2), 18–28.

Nordhaus, W. D. (2008). Baumol's diseases: a macroeconomic perspective. *The BE Journal of Macroeconomics*, 8(1).

Nordhaus, W. D. (2015). *Are We Approaching an Economic Singularity? Information Technology and the Future of Economic Growth*. Working Paper 21547, National Bureau of Economic Research.

Peretto, P. (1998). Technological change and population growth. *Journal of Economic Growth*, 3(4), 283–311.

Peretto, P. F. (2018). Robust endogenous growth. *European Economic Review*, 108, 49 – 77.

Porter, M. E. & Stern, S. (2000). *Measuring the ideas production function: Evidence from international patent output*. Working Paper 7891, National Bureau of Economic Research.

Ramondo, N., Rodríguez-Clare, A., & Saborío-Rodríguez, M. (2016). Trade, domestic frictions, and scale effects. *American Economic Review*, 106(10), 3159–84.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5), 1002–37.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5), S71–102.

Romer, P. M. (1994). The origins of endogenous growth. *Journal of Economic Perspectives*, 8(1), 3–22.

Romer, P. M. (2015). Mathiness in the theory of economic growth. *American Economic Review*, 105(5), 89–93.

Sedgley, N. & Elmslie, B. (2010). Reinterpreting the Jones critique: A time series approach to testing and understanding idea driven growth models with transitional dynamics. *Journal of Macroeconomics*, 32(1), 103–117.

Segerstrom, P. S. (1998). Endogenous Growth without Scale Effects. *American Economic Review*, 88(5), 1290–1310.

Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1), 65–94.

Sveikauskas, L. (1990). Productivity growth and the depletion of technological opportunities. *Journal of Productivity Analysis*, 1(4), 301–308.

Temple, J. (2003). The long-run implications of growth theories. *Journal of Economic Surveys*, 17(3), 497–510.

Ulku, H. (2007). R&D, innovation, and growth: evidence from four manufacturing sectors in OECD countries. *Oxford Economic Papers*, 59(3), 513–535.

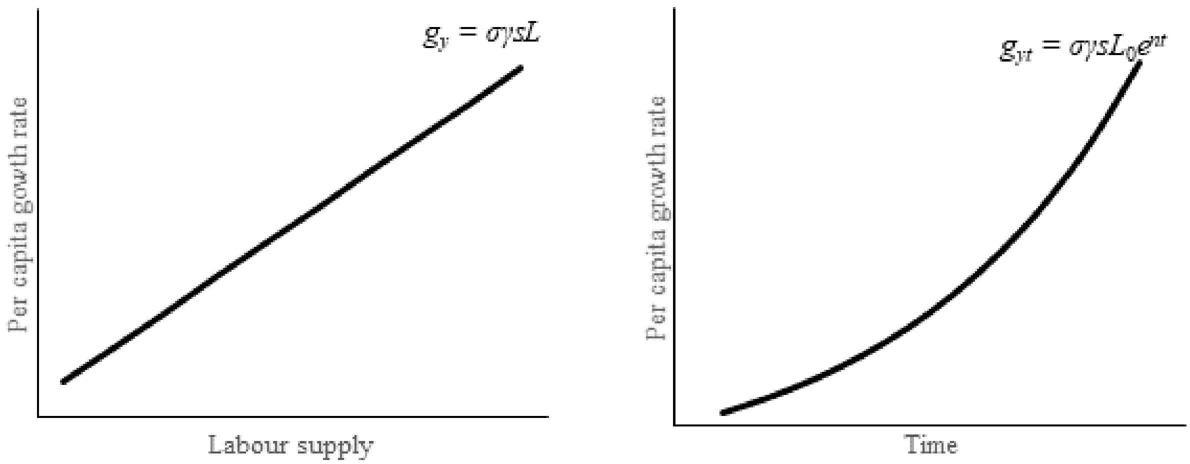
Venables, A. J. (1996). Equilibrium locations of vertically linked industries. *International Economic Review*, 37(2), 341–59.

Young, A. (1995). The tyranny of numbers: Confronting the statistical realities of the east asian growth experience. *The Quarterly Journal of Economics*, 110(3), 641–680.

Young, A. (1998). Growth without scale effects. *Journal of Political Economy*, 106(1), 40–63.

Zachariadis, M. (2003). R&D, innovation, and technological progress: a test of the Schumpeterian framework without scale effects. *Canadian Journal of Economics*, 36(3), 566–586.

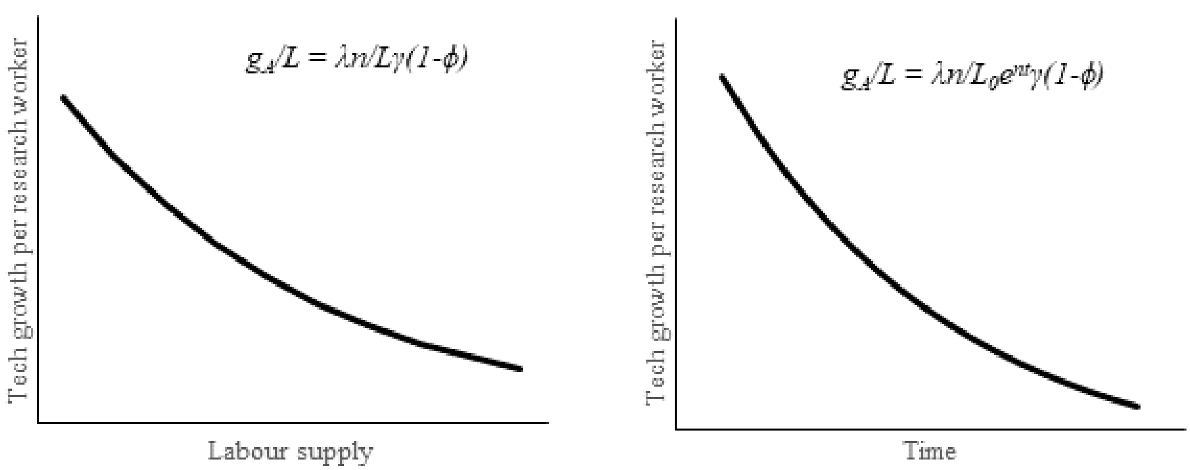
Zachariadis, M. (2004). R&D-induced Growth in the OECD? *Review of Development Economics*, 8(3), 423–439.



(a) Relationship between growth and population size

(b) Growth over time with population growing

Figure 1: The scale effect in first generation models



(a) Innovation productivity with a growing population

(b) Innovation productivity over time with a growing population

Figure 2: Diminishing innovation productivity in 'Semi'-endogenous growth models

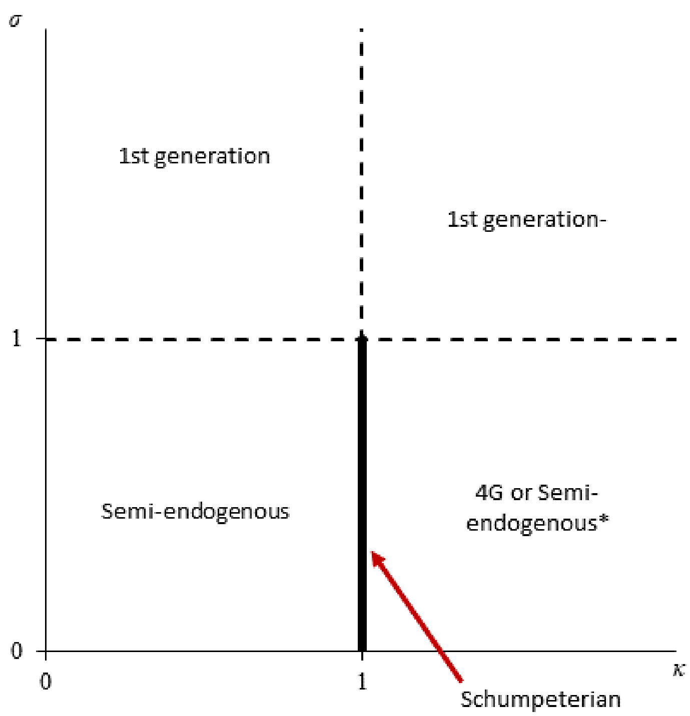


Figure 3: Parameter requirements of each generation of endogneous growth models

*The semi-endongeous case results for particular specifications in which either of two inequalities fails. See Peretto (2018, p. 56).

		Assumptions to remove scale effect				Additional assumptions	Linearity critique	
		Expanding variety in response to population	Expanding variety in response to vertical innovation	Diminishing returns to knowledge accumulation in aggregate	Diminishing returns to knowledge accumulation by the firm	Diminishing returns to knowledge in additional sectors	Growth	Population
Semi-endogenous growth	Jones (1995a)			✓				✓
	Kortum (1997)				✓			✓
	Segerstrom (1998)				✓			✓
	Li (2000)	✓			✓			✓
Schumpeterian endogenous growth	Young (1998)**	✓					✓	
	Howitt (1999)	✓				✓*	✓	
	Peretto (1998)	✓					✓	
	Dinopoulos & Thompson (1998)**	✓					✓	
4G endogenous growth	Peretto (2018)	✓	✓			?*	NA	NA

Table 1: Summary of endogenous growth models without scale effects and their assumptions

*Howitt (1999) includes diminishing returns to innovation in additional sectors, rather than knowledge acculululation. This achieves diminishing returns to population growth as identified by Jones (1995b), rather than to remove scale effects per se. Scale effects are removed by expanding product variety with population. This assumption would also be possible in Peretto's (2018) model.

** While proportional support for R&D has no effect on long run growth in these models, growth is still conisidered fully-endogenous because growth is supported by R&D subsidies that target (1) research intensity or (2) quality improvements, but not new varieties.

		Approach*			
		Idea production function		Contrast theories directly	
		Patents as the knowledge stock	Non-patent data as the knowledge stock	Long run trends	Co-integration tests
Support**	Semi-endogenous	Jaffe (1986); Furman et al. (2002); Luintel & Khan (2009); Barcenilla-Visús et al. (2014).	Hu & Mathews (2005); Hu & Mathews (2008).	Jones (1999); Jones (2002); [Ben] Jones (2009); [Ben] Jones (2010); Bloom et al. (2018).	
	Fully-endogenous (Schumpeterian or 4G)	Adams & Jaffe (1996); Porter & Stern (2000); Hu & Mathews (2005); Hu & Mathews (2008); Ulku (2007); Ang & Madsen (2015); Madsen (2016); Luintel & Khan (2017).	Furman et al. (2002); Furman & Hayes (2004); Abdih & Joutz (2006); Ang & Madsen (2011).	Griliches (1986); Griliches (1990); Sveikauskas (1990); Zachariadis (2003); Zachariadis (2004); Laincz & Peretto (2006); Ha & Howitt (2007); Ulku (2007); Madsen (2008); Madsen (2010); Madsen et al. (2010); Ang & Madsen (2011); Madsen (2016).	Ha & Howitt (2007) Madsen (2008) Sedgley & Elmslie (2010) Madsen et al. (2010) Ang & Madsen (2011)

Table 2: Summary of Empirical studies

* Studies may be shown more than once if multiple approaches were used.
** Support is categorized by the main conclusions of the article with respect to semi- or fully-endogenous growth (if given). For studies without conclusions related to the form of endogenous growth, the author's interpretation of the results is used to classify the article. Studies can appear more than once if results were mixed.

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