Appendix C Equilibrium for Firm 1, rivals and the entrant defined by constant n

Firm 1 will choose the autarky regime wherever network externalities are above the threshold $\alpha_1 > \frac{Q_C - Q_A}{Q_C + \sum_{j=2}^{n} b_j}$ where $Q_C = \sum_{j=2}^{n} q_i$ and $Q_A = \sum_{j=2}^{n} q_i$ refers to rivals total output in equilibrium under compatibility and autarky regimes respectively. Innovation affects the subsequent relative market shares so innovation also affects the regime chosen by Firm 1 and the counterfactual regimes that would apply if each bid were to fail to acquire the innovation. As a result, innovation can trigger potential changes in compatibility that affect auction outcomes and therefore also affect innovation incentives in the first place. Examining how innovation affects and is affected by compatibility requires extracting the variables directly impacted by innovation from the threshold. To keep the definition of n consistent under either acquisition or entry, let n refer to the number of incumbent firms such that entry requires $Q_C = \sum_{j=2}^{n+1} q_i = q_e + \sum_{j=2}^{n} q_i$ and $Q_A = \sum_{j=2}^{n+1} q_i = q_e + \sum_{j=2}^{n} q_i$, but under acquisition the definitions remain the same. The output equations per firm under compatibility and autarky can be rearranged to find the output of Firm 1 (q_1), the entrant (q_e) and rival incumbents (q_i) as well as aggregate output of rival incumbents ($\sum_{j=2}^{n} q_i$). This allows the threshold for Firm 1's compatibility decision to be specified for each alternate auction outcome: entry, included in Firm 1's output if Firm 1 acquires the innovation or included in rivals' output if another incumbent acquires the innovation. In order to accurately model outcomes with entry, acquistion, compatibility and autarky, the sections below recalculate equilibrium in terms of rival incumbent output. The threshold can then be reexamined on this basis.

Compatibility with entry

Rearranging output per firm under compatibility, with n now refering to the number of incumbent firms, the equations:

$$q_i = A_i + (\alpha - 1)\left(q_1 + q_e + \sum_{j=2}^{n} q_j\right) + \alpha\left(b_1 + b_e + \sum_{j=2}^{n} b_j\right) - c; \tag{1}$$

$$q_1 = \frac{A_1 + (\alpha - 1)\left(q_e + \sum_{j=2}^{n} q_j\right) + \alpha\left(b_1 + b_e + \sum_{j=2}^{n} b_j\right) - c}{2 - \alpha} \tag{1a}$$

$$q_e = \frac{(2 - \alpha) A_e + (\alpha - 1) A_1 + (\alpha - 1)\left(\sum_{j=2}^{n} q_j\right) + \alpha\left(b_1 + b_e + \sum_{j=2}^{n} b_j\right) - c}{3 - 2\alpha}; \textit{ and} \tag{1b}$$

$$\sum_{j=2}^{n} q_j = \frac{(3 - 2\alpha)\sum_{j=2}^{n} A_i + (n-1)(\alpha - 1)(A_1 + A_e) + (n-1)\left(\alpha\left(b_1 + b_e + \sum_{j=2}^{n} b_j\right) - c\right)}{(3 - 2\alpha)(\alpha - n\alpha + n) - 2(n-1)(\alpha - 1)^2}; \tag{1c}$$

now define equilibrium output with new entry under compatibility for Firm 1 (q_1), the entrant (q_e) and rival incumbents (q_i) as functions of the aggregate output of rival incumbents ($\sum_{j=2}^{n} q_i$).

Rearranging output per firm under autarky, with n now refering to the number of incumbent firms, the equations:

$$q_i = A_i + (\alpha - 1)\left(q_e + \sum_{j=2}^{n} q_j\right) + \alpha\left(b_e + \sum_{j=2}^{n} b_j\right) - q_1 - c; \tag{2}$$

$$q_1 = \frac{(2 - \alpha)(A_1 + \alpha b_1 - c) - \left(A_e + \alpha\left(b_e + \sum_{j=2}^{n} b_j\right) - c\right) - \sum_{i=2}^{n} q_i}{(2 - \alpha)^2 - 1}; \tag{2a}$$

$$q_e = \frac{(2 - \alpha)\left(A_e + (\alpha - 1)\sum_{j=2}^{n} q_j + \alpha\left(b_e + \sum_{j=2}^{n} b_j\right) - c\right) - \left(A_1 + \alpha b_1 - \sum_{i=2}^{n} q_i - c\right)}{(2 - \alpha)^2 - 1}; \textit{ and} \tag{2b}$$

$$
\begin{aligned}
\sum_{i=2}^{n} q_i \quad = \quad & \frac{\left((2-\alpha)^2-1\right)\left(\sum_{i=2}^{n} A_i+\alpha\left(b_e+\sum_{j=2}^{n} b_j\right)-c\right)((\alpha-1)(2-\alpha)+1)}{(2-\alpha)(3-2\alpha)} \\
& \times\left(A_e+\alpha\left(b_e+\sum_{j=2}^{n} b_j\right)-c\right)-\left(A_1+\alpha b_1-c\right)
\end{aligned}
\tag{2c}
$$

now define equilibrium output with new entry under autarky for Firm 1 (q_1), the entrant (q_e) and rival incumbents (q_i) as functions of the aggregate output of rival incumbents ($\sum_{j=2}^{n} q_i$).

Innovation and compatibility if Firm 1 acquires the innovation

If Firm 1 acquires the innovation, all of its output takes on the quality of the innovation ($A_1 = A_e$) and the installed base of the entrepreneur (b_e) is added to its installed base ($b_1 + b_e$). Rearranging compatible output if Firm 1 acquires the innovation, with n now refering to the number of incumbent firms, the equations:

$$
q_i = A_i + (\alpha-1)\left(q_1+\sum_{j=2}^{n} q_j\right)+\alpha\left(b_1+b_e+\sum_{j=2}^{n} b_j\right)-c; \tag{3}
$$

$$
q_1 = \frac{A_e+(\alpha-1)\left(\sum_{j=2}^{n} q_j\right)+\alpha\left(b_1+b_e+\sum_{j=2}^{n} b_j\right)-c}{(2-\alpha)} \;;\ and \tag{3a}
$$

$$
\sum_{j=2}^{n} q_j = \frac{\sum_{i=2}^{n} A_i + (n-1)\left(\alpha\left(b_1+b_e+\sum_{j=2}^{n} b_j\right)-c+(\alpha-1)A_e\right)}{n-\alpha n+1} \tag{3b}
$$

now define equilibrium output for Firm 1 (q_1) and rival incumbents (q_i) as functions of the aggregate output of rival incumbents ($\sum_{j=2}^{n} q_i$) if Firm 1 acquires the innovation under compatibility.

Rearranging autarkic output if Firm 1 acquires the innovation with n now refering to the number of incumbent firms, the equations:

$$
q_i = A_i + (\alpha-1)\sum_{j=2}^{n} q_j + \alpha\sum_{j=2}^{n} b_j - q_1 - c; \tag{4}
$$

$$
q_1 = \frac{A_e+\alpha\left(b_e+b_1\right)-\sum_{j=2}^{n} q_j - c}{2-\alpha};\ and \tag{4a}
$$

$$
\sum_{j=2}^{n} q_j = \frac{(2-\alpha)\sum_{i=2}^{n} A_i + (n-1)\left(\alpha(2-\alpha)\sum_{j=2}^{n} b_j - \alpha\left(b_e+b_1\right)-(3-\alpha)c-A_e\right)}{(2-\alpha)(\alpha-n\alpha+n)-(n-1)} \tag{4b}
$$

now define equilibrium output for Firm 1 (q_1) and rival incumbents (q_i) as functions of the aggregate output of rival incumbents ($\sum_{j=2}^{n} q_i$) if Firm 1 acquires the innovation under autarky.

Innovation and compatibility if a rival incumbent acquires the innovation

If a rival incumbent (Firm j) acquires the innovation all output of that rival takes on the quality of the innovation ($A_j = A_e$) and the installed base of the entrepreneur (b_e) is added to its installed base ($b_j + b_e$). Rearranging compatible output if a rival incumbent k acquires the innovation with n now refering to the number of incumbent firms, the equations:

$$
q_i = A_i + (\alpha-1)\left(q_1+q_k+\sum_{j=2,j\neq k}^{n} q_j\right)+\alpha\left(b_1+b_e+\sum_{j=2}^{n} b_j\right)-c; \tag{5}
$$

$$
q_1 = \frac{(2-\alpha)\left(A_1+\alpha b_1-c\right)-\left(A_e+\alpha\left(b_e+\sum_{j=2}^{n} b_j\right)-c\right)-\sum_{j=2,j\neq k}^{n} q_i}{(2-\alpha)^2-1}; \tag{5a}
$$

$$
q_k = \frac{(2-\alpha)A_e+(\alpha-1)A_1+(\alpha-1)\sum_{j=2,j\neq k}^{n} q_j+\alpha\left(b_1+b_e+\sum_{j=2}^{n} b_j\right)-c}{3-2\alpha};\ and \tag{5b}
$$

$$\begin{aligned}\textstyle\sum_{j=2,j\neq k}^{n} q_j &= \tfrac{(n-2)(\alpha-1)q_1+(n-2)(\alpha-1)q_k+\sum_{j=2,j\neq k}^{n} A_i}{(1-(n-2)(\alpha-1))} \\ &+\tfrac{(n-2)\alpha\left(b_1+b_e+\sum_{j=2}^{n} b_j\right)-(n-2)c}{(1-(n-2)(\alpha-1))}\end{aligned} \tag{5c}$$

now define equilibrium output for Firm 1 (q_1), non-acquiring rival incumbents (q_i) and the acquiring rival incumbent(q_k) as functions of the aggregate output of rival incumbents ($\sum_{j=2,j\neq k}^{n} q_i$) if a rival incumbent acquires the innovation under compatibility.

Rearranging auratkic output if a rival incumbent k acquires the innovation with n now refering to the number of incumbent firms, the equations:

$$q_i = A_i + (\alpha-1)\left(q_k + \sum_{j=2,j\neq k}^{n} q_j\right) + \alpha\left(b_1 + b_e + \sum_{j=2}^{n} b_j\right) - q_1 - c; \tag{6}$$

$$q_1 = \frac{A_e + \alpha\left(b_e + b_1\right) - \sum_{j=2,j\neq k}^{n} q_j - q_k - c}{2-\alpha};\ and \tag{6a}$$

$$q_k = \frac{(2-\alpha)\left(A_e + (\alpha-1)\sum_{j=2,j\neq k}^{n} q_j + \alpha\left(b_e + \sum_{j=2}^{n} b_j\right) - c\right) - \left(A_1 + \alpha b_1 - \sum_{j=2,j\neq k}^{n} q_i - c\right)}{(2-\alpha)^2 - 1};\ and \tag{6b}$$

$$\begin{aligned}\textstyle\sum_{j=2,j\neq k}^{n} q_j &= \textstyle\sum_{j=2,j\neq k}^{n} A_i + (n-2)(\alpha-1)\left(q_k + \sum_{j=2,j\neq k}^{n} q_j\right) \\ &+\alpha(n-2)\left(b_1 + b_e + \textstyle\sum_{j=2}^{n} b_j\right) - (n-2)\,q_1 - (n-2)\,c\end{aligned} \tag{6c}$$

now define equilibrium output for Firm 1 (q_1) and rival incumbents (q_i) as functions of the aggregate output of rival incumbents ($\sum_{j=2}^{n} q_i$) if a rival incumbent acquires the innovation under autarky.