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Abstract

This article considers the effect of a discrete entry barrier (i.e. an integer number of firms) in an endogenous growth model to draw conclusions about the relationship between contestability, innovation and growth. Sector-specific workers provide a tool for calibrating numerical examples. Sectors with lower entrepreneurial contestability have lower innovation and sectors characterized by Cournot oligopoly have lower innovation than sectors characterized by Bertrand. Wage inequality varies depending on the extent that the entry barrier is binding upon a marginal entrant. The model offers policy implications to support entrepreneurial entry, particularly in relatively small or isolated regional economies.

JEL classifications: O41; L13
Keywords: Innovation; contestability; Cournot; Bertrand; competition; endogenous growth

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Abstract

This article considers the effect of a discrete entry barrier (i.e. an integer number of firms) in an endogenous growth model to draw conclusions about the relationship between contestability, innovation and growth. Sector-specific workers provide a tool for calibrating numerical examples. Sectors with lower entrepreneurial contestability have lower innovation and sectors characterized by Cournot oligopoly have lower innovation than sectors characterized by Bertrand. Wage inequality varies depending on the extent that the entry barrier is binding upon a marginal entrant. The model offers policy implications to support entrepreneurial entry, particularly in relatively small or isolated regional economies.

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1 Introduction

The recent experience of the US economy is increasing market power (De Loecker and Eeckhout, 2017), concentration (Furman and Orszag, 2018) and rent seeking (Autor et al., 2017). Standard models of growth and innovation that assume free entry imply that market power is only temporary, as a reward for the cost and effort of developing and commercializing an innovation. If the market power that is won by developing technological improvements has increased, assuming free entry implies that potential entrants would also increase their efforts to develop innovations. But the contribution of innovation (Bloom et al., 2018; Gordon, 2016, 2018) and entrepreneurship (Decker et al., 2014, 2016, Hathaway and Litan, 2014; Guzman and Stern, 2016) to growth are declining along with shrinking dynamism in the labor market (Haltiwanger et al., 2013; Molloy et al., 2016). Free entry is an increasingly unreliable assumption for understanding innovation and growth in modern economies that are characterised by market power and concentration. Notably, real world markets are not the continuous functions assumed in growth models with infinitely small firms. Products are highly differentiated and firms compete with only a few rivals. Therefore, *standard assumptions of free entry are inadequate for understanding innovation and growth if markets are not entirely contestable.*

Innovative entry often requires a specific factor of production which represents a *structural* barrier and this factor attracts a premium for its scarcity. For example, Uber recently poached 40 researchers and scientists from Carnegie Mellon University offering “bonuses of hundreds of thousands of dollars and doubling the salaries to staff."[1] This problem may be exacerbated if geography and location limit access to the crucial factor required for entrepreneurial entry. Market liberalization has removed many regulatory and trade barriers, but other barriers to entry may still hamper innovation. If firms require a minimum viable scale[2] for innovative entry then the market may have capacity for only a few firms, particularly in small or isolated regions where the number of entrants may be limited by the resource base available and size of the market. Understanding the effect of market structure on entrepreneurship, innovation, growth and inequality is vitally important to developing policies that support long-run inclusive growth and arrest declining economic dynamism. On this basis, the relationship between barriers to entry and growth or innovation would be clearer if contestability itself is a parameter of market structure in models of growth and innovation. *Using discrete entry (i.e. only an integer number of firms is permitted) as a tool to model entry barriers, this article develops an understanding of how limited contestability affects innovation, can influence inequality and how entrepreneurship, innovation and growth are nuanced by the mode of competition.*

Empirical evidence examining the relationship between market structure and innovation suggests that the important factor is technological opportunity (Scherer, 1967; Levin et al., 1985; Hashmi, 2013), yet research has typically focused on competition (Aghion et al., 2009, 2005). Models of competition and innovation may overlook whether barriers to entry prevent markets from being contestable, by assuming free entry. Often endogenous growth models use a distance to frontier approach (Acemoglu et al., 2006; Aghion et al., 2009) that requires additional investment for entrants to match frontier technology. While entry costs are typically thought of as barriers to entry, theoretical models still assume *free* entry where investors expect to earn a return for this otherwise arbitrary additional investment required for entry. Economists have also long considered how rivalry affects innovation (Gilbert and Newbery, 1982; Fudenberg et al., 1983; Salant, 1984; Harris and Vickers, 1985; Reinganum, 1989). Innovations allow firms to deter entry by keeping ahead of rivals (Acemoglu and Akcigit, 2012; Acemoglu and Cao, 2015), Similarly, growth models imply that the threat from frontier entrants influences incumbent responses to escape entry or competition by innovating (Aghion et al., 2001, 2009; Acemoglu et al., 2006; Griffith et al., 2009). All of these models impose an essentially arbitrary cost for technology improvements, but otherwise consistently assume free entry. Such an assumption ignores the advice of Arrow (1962) that monopolists face a disincentive to innovate and disrupt the status quo (Shapiro, 2012).

I modify the endogenous growth model from Young (1998) by disaggregating utility into many sectors and allowing entry to a sector if a discrete marginal firm

[1] Source: Wall Street Journal (2015) Carnegie Mellon reels after Uber lures away researchers, http://on.wsj.com/1FnLNyW. Accessed November 2016.

[2] For a definition see Section 3.3 Horizontal Merger Guidelines issued by the U.S. Department of Justice and the Federal Trade Commission April 2, 1992 and revised April 8, 1997. Alternatively Section 9.3 of the current guidelines issued August 19, 2010, considers whether entry is sufficient.

is profitable. Yang and Heijdra's (1993) technique for solving the Dixit-Stiglitz model of monopolostic competition allows for market power by accounting for the price index effect. Disaggregation into many sectors similar to Venables (1999) allows for a segmented market structure in an otherwise aggregate model of growth. There is a limit on the factor of production required for entry in each sector and discrete entry enables only a limited number of firms rather than an entirely contestable market. The simplicity of this approach means that the critical factor of production becomes a measure of contestability that can be used for calibration, similar to Krugman's (1982) approach to studying comparative advantage or to the specific factor model of international trade (Jones, 1971; Samuelson, 1971). The model is completed analytically, but since results are not continuous functions, numerical examples with contestability as a calibration parameter are used to explore how innovation relates to competition, barriers to entry and inequality. The discrete entry approach is also validated by its use elsewhere. For example, Desmet and Parente (2014) also apply Yang and Heijdra's (1993) techniques to consider how specialized workers can block technology adoption that could substitute their input. Specific factors of production required for entry are generalised in the model by a single factor (labor), but the model could encompass any specific factor required for entry such as a natural resource, access to a network, sector specific skills, entrepreneurial skills, access to finance or collateral, or any other scarce factor of production that is necessary for market participation. In this way, the model becomes a framework to examine the impact on innovation and growth from a limitation on any specific factor of production required for entry.

As a result, sectors with less contestability experience greater rent seeking, less innovation and greater inequality. Intuitively, established firms are protected from the threat of a marginal entrant and only the competitve pressure of other market participants provides a weak incentive to develop innovations. Barriers to entry mean that workers in production are unable to become innovators who achieve entrepreneurial rewards, leading to greater inequality, less innovation and lower growth. Sectors characterized by Cournot oligopoly are less innovative than sectors characterized by Bertrand, but enable greater entry. Intuitively, firms in sectors characterized by Cournot invest less in innovation in order to withhold a greater share of profit. The effect of Cournot is mitigated because Cournot also reduces the innovation investment required for entry such that the scarce factor of production can be shared across a greater number of discrete firms, but the incentive provided by additional entry does not compensate entirely for the effect of Cournot. These findings are supported by numerical examples.

The article is set out as follows. Section 2 fully specifies the model including consumer preferences, innovation, the supply of labor and market entry. Subsequently, Section 3 derives equilibrium prices, wages and production, steady state rates of innovation in each sector and economy-wide growth rates. Section 4 describes a series of numerical examples to examine the relationship between contestability and innovation. Section 5 discusses the models findings and lastly, Section 6 provides some concluding remarks.

2 Model specification

This section fully specifies the model. For simplicity, consumption is modeled as a representative consumer with a taste for variety in each sector. There are many sectors (indexed by i) and a small discrete number of varieties (indexed by j) in each sector. A single variety j in sector i is referred to as variety i, j. In each period, the quality leading firm produces variety i, j using a sector specific labor supply as a monopolist competing with other differentiated varieties in sector i and, if the firm is going to continue production in the following period, the firm purchases a sufficient quality improvement from entrepreneurial workers to be the quality leading firm in that variety in the following period. Participating firms pay a competitive wage to all workers and entrepreneurs also receive a dividend from monopoly profits.

2.1 Barriers to entry

Prior to specifying the model, consider the concept of modeling barriers to entry. In an economic modeling sense, *free* entry can be thought of as an expectation that potential entrepreneurs will pursue profitable opportunities and enter profitable segments of the market. This potential for entry means a market is contestable, even if entrants face varying entry costs. While these costs are sometimes referred to as barriers to entry, a *structural barrier to entry* represents a barrier to even the potential for additional entry or contestability. With this understanding of barriers to entry, the existing literature may be inadequate to understand the relationship between contestability and innovation. Therefore, the model does not assume free entry by a continuous array of firms and instead uses a simple discrete entry barrier that only allows market participation by an integer number of firms that meet a requirement for positive profits and uses the techniques of Yang and Heijdra (1993) to incorporate firm responses to market power.

The discrete entry barrier involves two assumptions: (1) Only an integer number of firms may participate; and (2) that some specific factor of production is *required* as part of the cost of entry. It is this factor of production that pins down the number of participants in a each sector, such that once participation is determined, participating firms do not need to consider the threat of a marginal entrant. In the model here, the scarce factor of production is the sector specific labor supply. Innovations supplied by workers undertaking entrepreneurial effort is required for quality improvement and profitable entry. Entrepreneurs who find enough partners such that their collective entrepreneurial effort results in a sufficient quality improvement, receive a dividend as part of a firm's fixed cost, pinning down the number of firms to the number of entrepreneurial partnerships that achieve sufficient improvement.[3]

[3]Further discussion of barriers to entry and the techniques used in this paper are in an appendix.

2.2 Preferences

Constant elasticity of substitution (CES) utility by sector is nested in a Cobb-Douglas function with many sectors of only a few varieties each. The number of sectors is fixed, but the number of varieties in each sector is determined by sector parameters. The representative consumer has intertemporal preferences:

$$U = \sum_{t=0}^{\infty} \alpha^t \ln Q_t, \quad \alpha = 1/(1+\rho), \quad Q_t = \prod_{i=1}^{N} c_{i,t}^{\frac{1}{N}}. \tag{1}$$

where ρ represents the rate of time preference and Q_t is the Cobb-Douglas consumption of manufactured goods from N sectors in period t. Alternatively, Equation Q_t could be interpreted as a production function for a competitive final good sector which could serve as the numériare to aid empirical analysis. Varieties in different sectors are neither complements nor substitutes such that a change in the price of a variety in one sector has no effect on demand for varieties in other sectors and each sector has a constant expenditure share of $1/N$. Consumers have CES preferences between varieties (j) in the same sector (i):

$$c_{i,t} = \left[\sum_{j \in i} (A_{i,j,t} c_{i,j,t})^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad \sigma_i > 1 \forall i = 1, ..., N \tag{1a}$$

where $A_{i,j}$ represents the symmetric quality of variety j in sector i and the elasticity of substitution between varieties in the same sector is σ_i.

Intertemporal utility optimization implies the transversality condition and Euler equation $\frac{E_{t+1}}{E_t} = \frac{1+r}{1+\rho}$, where E_t is expenditure in period t and r is the rate of return on savings between period t and $t+1$. Rearranging gives $\frac{E_{t+1}}{1+r} = \frac{E_t}{1+\rho} = \alpha E_t$. Expenditure is normalized to $E_t = 1 \forall t$. The subscript t is suppressed where the time dimension is the same for all variables and its inclusion is unnecessary.

2.3 Labor

Following the approach of Krugman (1982) each industrial sector is assumed to have an exclusive labor supply L_i where workers inelastically provide one unit of labor per period. Workers undertake productive effort either as entrepreneurs or through employment in manufacturing. Workers are immobile between industry sectors, but mobile between firms and mobile between entrepreneurship and manufacturing. This assumption implies workers' skills are industry-specific and it is difficult for workers to re-skill for employment in other sectors. Labor supply and entrepreneurship pin down the number of participants per sector and labor can therefore be used to calibrate contestability. An equivalent assumption may be placed on any other specific factor of production required for entry but for simplicity the model assumes labor is the only factor of production. Industries with a large labor supply will have a greater ability for firms to enter and contest the market. This allows a direct comparison of the level of contestability and innovation, without altering demand parameters such as elasticities.

In each industrial sector, the labor supply is given by $L_{i,t} = \sum_{j\varepsilon n_{i,t},n_{i,t+1}} l_{i,j,t}\ \forall i = 1,...,N$, where $l_{i,j,t}$ is labor employed in entrepreneurship and manufacturing in period t by the firm producing variety i,j and $n_{i,t}$ and $n_{i,t+1}$ are the discrete number of firms in sector i in periods t and $t+1$ respectively. Both n_t and n_{t+1} are required since manufacturing workers produce current versions and entrepreneurs produce quality improvements for production in the following period.

Production involves a fixed labor cost of $F_{i,j,t-1}$ in the period prior to production and a constant marginal labor cost of β. For all sectors $i = 1,2,...,N$ and varieties $j = 1,2,...,n_i$, the labor required by each firm in period t is:

$$\begin{array}{lcll} l_{i,j,t} & = & 0 & if\, c_{i,j,t} = 0\, and\, c_{i,j,t+1} = 0; \\ l_{i,j,t} & = & \beta c_{i,j,t} & if\, c_{i,j,t} > 0\, and\, c_{i,j,t+1} = 0; \\ l_{i,j,t} & = & F_{i,j,t} & if\, c_{i,j,t} = 0\, and\, c_{i,j,t+1} > 0; \\ l_{i,j,t} & = & F_{i,j,t} + \beta c_{i,j,t} & if\, c_{i,j,t} > 0\, and\, c_{i,j,t+1} > 0. \end{array} \tag{2}$$

where $c_{i,j,t}$ is the period t production of variety i,j at the existing quality level that was developed in the previous period $t-1$. In the steady state the number of varieties in each sector is unchanging such that the labor requirement for the average firm in sector i is $l_{i,j,t} = F_{i,j,t} + \beta c_{i,j,t}$ and the total labor requirement in sector i is $L_{i,t} = \sum_{j\varepsilon n_{i,t+1}} F_{i,j,t} + \sum_{j\varepsilon n_{i,t}} \beta c_{i,j,t}$.

2.4 Technology, entrepreneurship and innovation

Innovation is modeled using the endogenous growth approach of Young (1998). Entrepreneurship is considered more than R&D because these workers perform both research and commercialization roles. In this model entrepreneurs are self-employed to develop and commercialize an innovative quality improvement in each period for a firm to earn a profit in the subsequent period as remuneration for the entrepreneur's effort. Entrepreneurs can only develop a quality improvement if there is capacity in the market for a discrete firm to produce that product in the following period. As workers inelastically provide one unit of labor per period, for entrepreneurs to achieve a greater quality improvement requires additional effort from entrepreneurial partners. While the firm earns a monopoly profit, it is shared between the entrepreneurial workers who developed the quality improvement as a dividend in addition to the competitive wage. Therefore, entrepreneurs face a trade-off between sharing profits with additional partners or allowing additional rival firms to enter the market and diminish profit. For the purposes of this model, workers can only undertake effort in either entrepreneurship or manufacturing.[4]

The entrepreneurial labor requirement to achieve the targeted quality level $A_{i,j,t}$ for variety i,j and the fixed cost incurred in the previous period $t-1$ for production

[4]As necessary for modelling purposes, this definition of entrepreneurship, excludes traditional self-employment. To compare to the model, traditional self-employment could be thought of as employment by a firm where the firm is owned by the worker and the worker undertakes effort in both entrepreneurship and manufacturing.

in period t is:

$$F_{i,j,t-1}(A_{i,j,t},\bar{A}_{i,j,t-1}) = \begin{cases} \gamma e^{\eta A_{i,j,t}/\overline{A}_{i,j,t-1}} & \text{if } A_{i,j,t} \geq \overline{A}_{i,j,t-1} \\ \gamma e^{\eta} & \text{otherwise} \end{cases} . \quad (3)$$

The parameters γ and η are constants used for calibration. $\overline{A}_{i,j,t-1}$ is an index of technological opportunity, representing the intertemporal spillover of knowledge to entrepreneurs developing improvements in variety i,j and innovations replace obsolete versions. The technological opportunity index is simply the highest existing quality level for variety i,j. Each variety is symmetric within its sector such that all varieties in the same sector have the same quality level. The cost of innovation can be thought of as two components: a commercialization cost of γe^{η} irrespective of quality improvement and a research cost of $\gamma e^{\eta A_{i,t}/\bar{A}_{i,j,t-1}} - \gamma e^{\eta}$.

Dividends from monopoly profits may provide entrepreneurs with greater earnings than manufacturing workers as discrete entry creates a barrier to employment as entrepreneurs, with manufacturing wages falling until the labor market clears. In models that extend this approach to other critical factors of production, this premium would accrue to whomever controls the relevant factor of production required for entry.[5]

To facilitate the potential for new varieties not produced in the previous period and to maintain symmetry, if there is capacity for an additional profitable firm, it is assumed that the index of technological opportunity is an average of quality levels for its sector: $\overline{A}_{i,j,t-1} = \frac{\sum_{j \in n_{i,t-1}} A_{i,j,t-1}}{n_{i,t-1}}$. Alternatively, if an incumbent acquired an innovation that was an insufficient improvement (for any arbitrary reason) such that a marginal discrete firm could profitably enter, the new disequilibrium would result in negative profits for some firms because entrepreneurs would have to fund the market clearing manufacturing wage out of their own wages. Therefore, some entrepreneurial workers would prefer higher incomes by offering their labor in manufacturing rather than entrepreneurship. The disequilibrium would be corrected when an unprofitable firm would inevitably exit the market as those workers returned to manufacturing. This characteristic can be thought of as the market having partially free entry, but the extent of contestability is determined by the availability of the entry-limiting factor of production. It is this mechanism that blocks the marginal entrant and limits market participation to an integer number of firms.

3 Equilibrium and the steady state

Equilibrium follows from optimization of CES sectors using techniques adapted from Yang and Heijdra (1993). All model variables are solved as functions of the number of firms and subsequently the number of entrants is solved and defined.

[5]More detailed models could also include strategies to control the critical factor of production, such as forming a cartel or lobbying for protective regulations.

This completes the model. As a result of discrete entry, profits are positive, but extracted by entrepreneurs appropriating all profit in the entrepreneurial "dividend" rather than by competitive pressure under a free entry condition. Usually in CES models, assuming free entry means both Cournot and Bertrand competition yield the same results, because the function is continuous or the number of firms is assumed to be very large. However, with discrete entry, outcomes differ by the mode of competition because each firm is not infinitesimally small.

3.1 Equilibrium

Consumers allocate expenditure across sectors and varieties subject to the budget constraint $\sum_{i \in N} P_i c_i \leq E$, where P_i is the price index of sector i (to be defined in Equation 4a), c_i is demand for all varieties in sector i (defined by Equation 1a) and E is expenditure. From Cobb-Douglas utility between sectors, expenditure per sector is $c_i P_i = \frac{E}{N}$, such that the consumer spends a $\frac{1}{N}$ share of her expenditure on varieties in each sector i. Utility optimization finds that the direct demand function (for determining equilibrium in a sector characterized by Bertrand oligopoly) for each variety i, j is given by:

$$c_{i,j} = \frac{E}{N} A_{i,j}^{\sigma_i - 1} P_{i,j}^{-\sigma_i} P_i^{\sigma_i - 1} \tag{4}$$

where $P_{i,j}$ and $A_{i,j}$ are the price and quality level of variety i, j respectively. P_i is the sector i index of price and quality defined by the budget constraint and derived as:

$$P_i = \left[\sum_{j \in i,} A_{i,j}^{\sigma_i - 1} P_{i,j}^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}}. \tag{4a}$$

Alternatively, utility optimization finds that the inverse demand function (for determining equilibrium in a sector characterized by Cournot oligopoly) for each variety i, j is given by:

$$P_{i,j} = A_{i,j}^{\frac{\sigma_i - 1}{\sigma_i}} c_{i,j}^{\frac{-1}{\sigma_i}} \frac{E}{N} \left[c_i^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{-1}, \tag{5}$$

where c_i is a consumption index defined byconsumer preferences (Equation 1a).

The earnings in period $t-1$ for each entrepreneur in sector i who developed the current quality level $A_{i,j,t}$ is

$$y_{i,E,t-1} = w_{i,t-1} + \frac{d_{i,t}}{(1 + r_t)}$$

where $w_{i,t-1}$ is the sector i wage of workers in period $t-1$ in order to clear the sector i labor market and $d_{i,t}$ is the sector specific dividend for entrepreneurs received in the following period.

The usual free entry condition, which implies that profits diminish to zero due to the competitive pressure of free entry, does not apply. In contrast to standard models with free or continuous entry, with the discrete entry barrier, firms permitted entry may make positive profit but all profit is paid to entrepreneurs through a dividend, over and above the standard manufacturing wage paid to all workers, including entrepreneurs. The free entry condition is modified to an alternative "entrepreneurial dividend condition" such that *entrepreneurs appropriate all profit and dividends are always positive*. Dividends are determined by the firm's profit function:

$$\pi_{i,j,t} = \frac{(P_{i,j,t} - \beta w_{i,t})\, c_{i,j,t}}{(1+r_t)} - w_{i,t-1} F_{i,j,t-1} = \frac{d_{i,t}}{(1+r_t)} F_{i,j,t-1} \geq 0, \tag{5a}$$

subject to demand and discrete entry. The initial entrepreneur determines whether to hire further entrepreneurs in order to increase effort to achieve a greater quality improvement. The optimization decision requires that these initial entrepreneurs attempt to maximize their income from both wages and dividends. Rearranging the firm's profit function (Equation 5a) and substituting entrepreneurial earnings, entrepreneurs make innovation and production choices that maximize earnings such that the firms output, wage, innovation and pricing decisions are governed by the optimization problem:

$$\max y_{i,E,t-1} = \frac{(P_{i,j,t} - \beta w_{i,t})\, c_{i,j,t}}{(1+r_t)\, F_{i,j,t-1}} \tag{5b}$$

subject to market demand.

Workers will choose to be entrepreneurs if dividends are positive and an entrepreneurship opportunity is available either as a partner to develop an improvement in an existing variety or by creating a new variety if there is capacity for a profitable marginal firm. If no entrepreneurial opportunity is available, remaining workers are employed in manufacturing to clear the labor market.

3.1.1 Cournot oligopoly

In sectors characterized by Cournot oligopoly, entrepreneurs maximize income subject to inverse demand (Equation 5) by choosing output and quality improvement on the basis of consumers' willingness to pay for that quantity and quality in order to clear the market. Differentiating Equation 5b with respect to the decision variables of output and quality respectively, the first order conditions for maximizing entrepreneurial income are given by: $\frac{\partial y_{i,E,t-1}}{\partial c_{i,j,t}} = \frac{P_{i,j,t}-\beta w_{i,t}}{(1+r_t)F_{i,j,t-1}} + \frac{c_{i,j,t}}{(1+r_t)F_{i,j,t-1}} \frac{\partial P_{i,j,t}}{\partial c_{i,j,t}} = 0$ and $\frac{\partial y_{i,E,t-1}}{\partial A_{i,j,t}} = \frac{c_{i,j,t}}{(1+r_t)} \left(\frac{\frac{\partial P_{i,j,t}}{\partial A_{i,j,t}} F_{i,j,t-1} - \left(P_{i,j,t} - \beta w_{i,t}\right) \frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}}{\left(F_{i,j,t-1}\right)^2} \right) = 0$. Rearranging the first of these first order conditions gives equilibrium Cournot prices:

$$P_{i,j,t} = \frac{1}{\left(1 + \frac{c_{i,j,t}}{P_{i,j,t}} \frac{\partial P_{i,j,t}}{\partial c_{i,j,t}}\right)} \beta w_{i,t} \tag{6}$$

where $\frac{c_{i,j,t}}{P_{i,j,t}}\frac{\partial P_{i,j,t}}{\partial c_{i,j,t}}$ represents the elasticity of quanity demanded with respect to price, evaluated using inverse demand. Symmetry in each sector means the inverse demand function (Equation 5) for variety i,j can also be written as $P_{i,j} = \frac{E}{N} A_{i,j}^{\frac{\sigma_i-1}{\sigma_i}} c_{i,j}^{\frac{-1}{\sigma_i}} \left(A_{i,j}^{\frac{\sigma_i-1}{\sigma_i}} c_{i,j}^{\frac{\sigma_i-1}{\sigma_i}} + (n_i-1) A_{i,k}^{\frac{\sigma_i-1}{\sigma_i}} c_{i,k}^{\frac{\sigma_i-1}{\sigma_i}} \right)$, where variety i,k represents each of the other symmetrical varieties in sector i. Differentiating this inverse demand function with respect to consumption of variety i,j, simplifying and substituting the differential and the inverse demand function to find the elasticity of substitution, the equilibrium price of variety i,j is evaluated as a function of the number of entrants by substituting the elasticity into Equation 6:

$$P_{i,j}(n_i) = \left(\frac{n_i \sigma_i}{(n_i-1)(\sigma_i-1)} \right) \beta w_i, \qquad n_i \geq 2. \tag{6a}$$

As n_i increases, the price of a firm's own variety has less effect on the industry index until the familiar CES pricing rule is reached $P_{i,j} = \beta w_i \left(\frac{\sigma_i}{\sigma_i - 1} \right)$.

In the case of $n_i = 1$ the firm would receive the same revenue irrespective of price due to Cobb-Douglas utility between sectors because consumers allocate a specific portion of expenditure on that sector. This limitation therefore requires that $n \geq 2$. To include the $n = 1$ scenario would require the upper-level of the utility function to be an alternative such as a CES function. For simplicity, the Cobb-Douglas version is used to aid intuition and there is no loss of generality for the conclusions drawn because this article is concerned with situations other than pure monopoly.

3.1.2 Bertrand oligopoly

In sectors characterized by Bertrand oligopoly, entrepreneurs maximize income subject to direct demand (Equation 4) by choosing price and quality on the basis of quantity demanded at that price and quality in order to clear the market. Differentiating Equation 5b with respect to the decision variables of price and quality respectively, the first order conditions are given by: $\frac{\partial y_{i,E,t-1}}{\partial P_{i,j,t}} = \frac{c_{i,j,t}}{(1+r_t)F_{i,j,t-1}(A_{i,j,t})} + \frac{(P_{i,j,t} - \beta w_{i,t})}{(1+r_t)F_{i,j,t-1}} \frac{\partial c_{i,j,t}}{\partial P_{i,j,t}} = 0$ and $\frac{\partial y_{i,E,t-1}}{\partial A_{i,j,t}} = \frac{(P_{i,j,t} - \beta w_{i,M,t})}{(1+r_t)} \left(\frac{\frac{\partial c_{i,j,t}}{\partial A_{i,j,t}} F_{i,j,t-1} - c_{i,j,t} \frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}}{(F_{i,j,t-1})^2} \right) = 0$. Rearranging the first of these first order conditions gives the equilibrium Bertrand price rule:

$$P_{i,j,t} \left(\frac{c_{i,j,t}}{P_{i,j,t}} \frac{1}{\frac{\partial c_{i,j,t}}{\partial P_{i,j,t}}} + 1 \right) = \beta w_{i,t} = \beta w_{i,t} \tag{7}$$

where $\frac{c_{i,j,t}}{P_{i,j,t}} \frac{1}{\frac{\partial c_{i,j,t}}{\partial P_{i,j,t}}}$ is the elasticity of demand with respect to price, evaluated using direct demand. While it seems familiar with Cournot prices, elasticities are not constant with a discrete number of firms, despite using CES demand. With discrete firms and symmetry in each sector, the direct demand function can be written as $c_{i,j}(n_i) = \frac{E}{N} A_{i,j}^{\sigma_i - 1} P_{i,j}^{-\sigma_i} \left[P_{i,j}^{1-\sigma_i} A_{i,j}^{\sigma_i - 1} + (n_i - 1) P_{i,k}^{1-\sigma_i} A_{i,k}^{\sigma_i - 1} \right]^{-1}$. Differentiating this direct demand function with respect to the price of variety i, j, simplifying and substituting the differential and the direct demand function to find the elasticity of substitution, the equilibrium price of variety i, j is evaluated as a function of the number of entrants by substituting the elasticity into Equation 7:

$$P_{i,j}(n) = \left(\frac{\sigma_i n_i - (\sigma_i - 1)}{(n_i - 1)(\sigma_i - 1)} \right) \beta w_i, \qquad n \geq 2. \tag{7a}$$

As with Cournot, when n_i increases, the price of a firm's own variety has less effect on the industry index until the familiar CES pricing rule $P_{i,j} = \beta w_i \left(\frac{\sigma_i}{\sigma_i - 1} \right)$ is also reached with Bertrand. The limitation that $n \geq 2$ still applies due to Cobb-Douglas preferences between sectors.

3.1.3 Labor market clearing for manufacturing

The labor used in manufacturing in sector i simply equals the number of units consumed multiplied by the marginal labor cost per unit (β). Dividing consumer expenditure in sector i by the symmetrical price per unit and multiplying by β finds the labor requirement in sector i is $l_{i,m,t} = \frac{E_t}{NP_{i,j,t}} \beta$. labor employed as entrepreneurs in period t equals the number of entrants in the coming period multiplied by the symmetric investment in quality improvement per firm, $l_{i,I,t} = n_{i,t+1} F_{i,j,t}$. Labor market clearing requires that the total specialized sectoral labor supply is employed. Total labor in each sector is therefore equal to $L_{i,t} = n_{i,t+1} F_{i,j,t} + \frac{E_t}{NP_{i,j,t}} \beta$. It is assumed that wages are not affected by the market power of employers.[6] However, wages are affected by competition in product markets as this determines workers' alternative employment opportunities as entrepreneurs.

Substituting Cournot prices finds that the labor employed in sector i, characterized by Cournot oligopoly, is given by: $L_{i,t} = n_{i,t+1} F_{i,t} + \frac{E_t (n_{i,t} - 1)(\sigma_i - 1)}{N n_{i,t} \sigma_i w_{i,t}}$. Solving for Cournot manufacturing wages gives:

$$w_{i,t} = \frac{E_t (n_{i,t} - 1)(\sigma_i - 1)}{N n_{i,t} \sigma_i (L_{i,t} - n_{i,t+1} F_{i,t})}. \tag{8}$$

Alternatively substituting Bertrand prices finds that the labor employed in sector i, characterized by Bertrand oligopoly, is given by $L_{i,t} = n_{i,t+1} F_{i,t} + \frac{E_t (n_{i,t} - 1)(\sigma_i - 1)}{N(\sigma_i n_i - (\sigma_i - 1)) w_{i,t}}$. Rearranging, Bertrand manufacturing wages are given by:

$$w_{i,t} = \frac{E_t (n_{i,t} - 1)(\sigma_i - 1)}{N(\sigma_i n_i - (\sigma_i - 1))(L_{i,t} - n_{i,t+1} F_{i,t})}. \tag{9}$$

[6] Of course, it is possible that workers with very specific skills in an industry with only a few employers could face non-competitive wages (Bhaskar et al., 2002), but monopsony pressure on the labor market is beyond the scope of this article.

3.2 Innovation

After manufacturing costs, all remaining future profits are paid in dividends to entrepreneurs in return for their efforts to develop the innovation. Workers are mobile between employment in manufacturing or entrepreneurship and there are no barriers to workers offering their labor for manufacturing. However, workers face a barrier to employment as an entrepreneur since opportunities either require a partnership with additional entrepreneurs or the capacity for an additional firm. Therefore, dividends must be positive, such that entrepreneurs' earnings are never less than the wages of manufacturing workers, because workers employed as entrepreneurs would switch to the manufacturing role if dividends were negative, but the reverse process may not be possible.

In a significant departure from Yang and Heijdra (1993), these techniques are adapted to also determine innovation. If the market is characterized by Cournot, it is assumed that profit maximization involves the joint optimization of quality improvement and output given consumers' willingness to pay for higher quality products and to clear the market. Alternatively, if the market is characterized by Bertrand, it is assumed that profit maximization involves the joint optimization of quality improvement and prices based on consumers' quantity demanded in response to higher quality products and proposed prices.

3.2.1 Cournot innovation

Rearranging the second of the first order conditions for maximizing entrepreneurial income in a market characterized by Cournot oligopoly and dividing both sides by $A_{i,j,t}$ gives $\frac{F_{i,j,t-1}}{A_{i,j,t}} \frac{1}{\frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}} = \frac{\left(P_{i,j,t} - \beta w_{i,t}\right)}{A_{i,j,t}} \frac{1}{\frac{\partial P_{i,j,t}}{\partial A_{i,j,t}}}$. As in Young (1998) entrepreneurs select a quality improvement where the elasticity of the research cost with respect to quality equals the elasticity of demand with respect to quality. However, with a few discrete firms, the firm takes into account the impact of its own quality improvement on the industry index. Differentiating Equations 3 and inverse demand, substituting the differentials into the above, evaluating $\beta w_{i,t}$ using Cournot prices and substituting inverse demand and the cost of innovation, the entrepreneur's income-maximizing quality-improvement target is:

$$A_{i,j,t} = \overline{A}_{i,j,t-1} \frac{(\sigma_i - 1)(n_{i,t} - 1)}{\eta\,(n_{i,t} + (\sigma_i - 1))}. \tag{10}$$

As in Young (1998) parameters are assumed such that there is always growth in the fully contestable market. That is $\frac{\sigma_i - 1}{\eta} > 1$.

It is possible that profit maximization would result in no growth in the steady state because $\frac{(\sigma_i-1)(n_{i,t}-1)}{\eta(n_{i,t}+(\sigma_i-1))} \leq 1$ for some low levels of n_i. Solving for n_i, growth will not occur if $n_{i,t} \leq \frac{(\eta+1)(\sigma_i-1)}{(\sigma_i-1)-\eta}$. That is, participation in the market at the minimum fixed cost deters the marginal firm, even without developing a quality improvement. In this case, the quality target is the existing quality level $\overline{A}_{i,j,t-1}$.

Substituting the quality target in Equation 10 into the innovation function (Equation 3), the entrepreneurial effort required per firm is:

$$F_{i,j,t-1} = \begin{cases} \gamma e^{\frac{(\sigma_i-1)\left(n_{i,t}-1\right)}{n_{i,t}+\left(\sigma_i-1\right)}} & \text{for}\, n_{i,t} > \frac{(\eta+1)(\sigma_i-1)}{(\sigma_i-1)-\eta} \\ \gamma e^{\eta} & \text{otherwise.} \end{cases} \tag{10a}$$

As n_i increases, the quality improvement target and investment in innovation tends towards the results in the continuous free entry model in Young (1998).

Substituting the research cost into Equation 5b and taking the revenue of all firms in sector i as symmetric, the income of entrepreneurs in period $t-1$ from both wages and future dividends in sectors characterized by Cournot competition is equal to:

$$y_{i,E,t-1} = \begin{cases} \frac{(n_{i,t}+\sigma_i-1)\alpha E_{t-1}}{Nn_{i,t}^2\sigma_i\gamma e^{\frac{(\sigma_i-1)\left(n_{i,t}-1\right)}{n_{i,t}+\left(\sigma_i-1\right)}}} & \text{for}\, n_{i,t} > \frac{(\eta+1)(\sigma_i-1)}{(\sigma_i-1)-\eta} \\ \frac{(n_{i,t}+(\sigma_i-1))\alpha E_{t-1}}{Nn_{i,t}^2\sigma_i\gamma e^{\eta}} & \text{otherwise.} \end{cases} \tag{11}$$

3.2.2 Bertrand innovation

Rearranging the second of the first order conditions for maximizing entrepreneurial income in a market characterized by Bertrand oligopoly and dividing both sides by $A_{i,j,t}$ gives $\frac{F_{i,j,t-1}}{A_{i,j,t}}\frac{1}{\frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}} = \frac{c_{i,j,t}}{A_{i,j,t}}\frac{1}{\frac{\partial c_{i,j,t}}{\partial A_{i,j,t}}}$. Differentiating Equations 3 and direct demand, substituting the differentials into the above and rearranging, the entrepreneur's earnings-maximizing quality target is:

$$A_{i,j,t} = \overline{A}_{i,j,t-1}\frac{(\sigma_i-1)\,(n_{i,t}-1)}{\eta n_{i,t}}. \tag{12}$$

For $\sigma_i > 2\eta+1$, Bertrand competition results in growth for any level of $n_i \geq 2$. For $1 < \sigma_i < 2\eta+1$, the supply of labor must be large enough that $n_i > \frac{\sigma_i-1}{(\sigma_i-1)-\eta}$ for growth to occur. Therefore, the entrepreneurial effort required per firm is:

$$F_{i,j,t-1} = \begin{cases} \gamma e^{\frac{(\sigma_i-1)\left(n_{i,t}-1\right)}{n_{i,t}}} & \text{for}\, n_{i,t} > \frac{\sigma_i-1}{(\sigma_i-1)-\eta} \\ \gamma e^{\eta} & \text{otherwise.} \end{cases} \tag{12a}$$

Similar to Cournot innovation, as n_i increases, the innovation target and investment in innovation tends toward the competitive level found in Young (1998).

Substituting the cost of innovation into Equation 5b and taking the revenue of all firms in sector i as symmetric, the income of entrepreneurs in period $t-1$ from both wages and future dividends in sectors characterized by Bertrand oligopoly is:

$$y_{i,E,t-1} = \begin{cases} \frac{\alpha E_{t-1}}{N(\sigma_i n_{i,t}-(\sigma_i-1))\gamma e^{\frac{(\sigma_i-1)\left(n_{i,t}-1\right)}{n_{i,t}}}} & \text{for } n_{i,t} > \frac{\sigma_i-1}{(\sigma_i-1)-\eta} \\ \frac{\alpha E_{t-1}}{N(\sigma_i n_{i,t}-(\sigma_i-1))\gamma e^{\eta}} & \text{otherwise.} \end{cases} \quad (13)$$

3.2.3 Endogenous variety

Workers will enter the labor market as entrepreneurs, either with partners in an existing variety or a new variety, until adding additional entrepreneurial effort or a marginal firm would result in entrepreneurial income falling below the manufacturing wage.

Substituting the Cournot manufacturing wage and entrepreneurial income (Equations 8 and 11 respectively), the positive dividend requirement can be rearranged to $L_{i,t} \geq \left(\frac{(n_{i,t}-1)(\sigma_i-1)n_{i,t+1}^2}{n_{i,t}(n_{i,t+1}+\sigma_i-1)\alpha} + n_{i,t+1}\right) F_{i,t}$. In the steady state, the number of firms is unchanging such that the largest integer $n_i \geq 2$ that satisfies

$$L_i \geq n_i \left(\frac{(n_i-1)(\sigma_i-1)}{(n_i+\sigma_i-1)\alpha} + 1 \right) F_i \quad (14)$$

implicitly defines the steady state number of firms in a sector characterized by Cournot competition. The Cournot model is now complete. The n_i which satisfies Equation 14 can be used to solve for any variable in Cournot competition using equations 6a, 8, 10 and 11.

Substituting Bertrand manufacturing wages and entrepreneurial income (Equations 9 and 13), the positive dividend requirement can be rearranged such that $L_{i,t} \geq \left(\frac{(n_{i,t}-1)(\sigma_i-1)(\sigma_i n_{i,t+1}-(\sigma_i-1))}{\alpha(\sigma_i n_{i,t}-(\sigma_i-1))} + n_{i,t+1}\right) F_{i,t}$. With a constant number of firms in the steady state, the largest integer $n_i \geq 2$ that satisfies

$$L_i \geq \frac{(n_i-1)(\sigma_i-1)+\alpha n_i}{\alpha} F_i \quad (15)$$

implicitly defines the steady state number of firms in a sector characterised by Bertrand. In the non-growth scenario for Bertrand competition, the number of firms can be explicitly defined as the largest integer $n_i \geq 2$ which satisfies:

$$n_i \leq \frac{\alpha L_i + (\sigma_i-1)\gamma e^{\eta}}{((\sigma_i-1)+\alpha)\gamma e^{\eta}} \quad (15a)$$

The Bertrand model is now also complete. The n_i which satisfies Equation 15 (or for $n_i < \frac{\sigma_i-1}{(\sigma_i-1)-\eta}$ also satisfies Equation 15a) can be used to solve for any variable in Bertrand competiton using equations 7a, 9, 12 and 13.

3.2.4 Cournot versus Bertrand Innovation

In comparable markets with the same parameters, the difference between quality targets for sectors characterized by Cournot or Bertrand oligopoly converges to zero as the number of firms tends to infinity. Similarly, Cournot and Bertrand models converge towards the equal outcomes as σ tends toward one. However, with the discrete entry requirement used to model limited contestability, the relationship between innovation targets for Cournot and Bertrand competition is not so clear, given that varieties are substitutes ($\sigma > 1$). A direct comparison between the quality targets cannot be made because the number of firms is implicitly defined by the size of each specialized labor type L_i and the number of firms differs between Cournot and Bertrand markets. This section seeks to establish and understand the comparison of innovation in sectors characterized by Cournot and Bertrand oligopoly.

Let all parameters be equal in two sectors characterized by Cournot and Bertrand oligopoly. Examine the difference between the two quality targets for Bertrand and Cournot sectors given in Equations 12 and 10 respectively, with n_B and n_C describing the number of firms in Bertrand and Cournot sectors:

$$A_{Diff} = A_{Bertrand} - A_{Cournot} = \overline{A}_{i,j,t-1}\frac{(\sigma_i - 1)}{\eta}\left(\frac{(n_B - 1)}{n_B} - \frac{(n_C - 1)}{(n_C + (\sigma_i - 1))}\right). \tag{16}$$

Setting this difference equal to zero and rearranging finds that in order for the difference to be zero, $\sigma_i = 1$ or $\sigma_i = \frac{n_C-1}{n_B-1}$. As the first of these options breaches the assumption that varieties in the same sector are substitutes ($\sigma_i > 1$), it can be seen that the difference in quality targets is only zero if the elasticity is equal to the ratio of $n_C - 1$ to $n_B - 1$. Now let $\sigma_i = \frac{n_C-1}{n_B-1} + \varepsilon$. By substituting into the above and solving for ε, the characteristics of ε describe the difference between quality targets for sectors defined by Bertrand and Cournot such that a positive ε implies that a sector with Bertrand oligopoly sets a higher quality target than a sector with Cournot oligopoly. Assuming that a Bertrand sector has the higher quality target so the difference between the two quality targets for Cournot and Bertrand sectors is positive, substituting and rearranging finds the two solutions for ε are both positive:

$$\begin{gathered}\overline{A}_{i,j,t}\frac{\left(\frac{n_C-1}{n_B-1}+\varepsilon-1\right)}{\eta} > 0 \; or \; \frac{(n_B-1)}{n_B} - \frac{(n_C-1)}{\left(n_C+\left(\frac{n_C-1}{n_B-1}+\varepsilon-1\right)\right)} > 0 \\ \varepsilon > \frac{n_B-n_C}{n_B-1} > 0 \; or \; \varepsilon > 0\end{gathered} \tag{17}$$

Assuming that a Cournot sector has a higher quality target requires that ε be negative.

The variables n_C and n_B are not independent because they are both implicitly defined by the same parameters according to equations 14 and 15. Making these definitions equal (based on the same L_i, thereby holding aside the integer requirement on n), substituting $n_B = \frac{1}{\sigma_i}(n_C + \sigma_i - 1)$ and rearranging, finds that $\sigma_i = \frac{n_C-1}{n_B-1}$

is only possible when:

$$\frac{(n_C-1)(\sigma_i-1)+\alpha(n_C+\sigma_i-1)}{\sigma_i\alpha} = n_C\left(\frac{(n_C-1)(\sigma_i-1)}{(n_C+(\sigma_i-1))\alpha}+1\right)$$
$$\sigma_i = 1, \tag{18}$$

As a result, ε must be positive because varieties are substitutes ($\sigma_i > 1$) such that it can be concluded that participating firms in a sector characterized by Bertrand oligopoly always set a higher quality target than participants in a comparable sector characterized by Cournot oligopoly for all possible calibrations. The intuitive explanation for this result is that firms in a Cournot market withhold investment compared to a Bertrand market. In doing so, Cournot firms allow additional competition to enter the market under the discrete entry barrier, but this competitive pressure never compensates for the effect of Cournot.

3.3 Steady-state growth

The measure of growth for the economy as a whole is the rate at which total output increases. In this model, total output is made up of two components: production of manufactured goods and production of quality-improving innovations. Output increases at the rate that real incomes increase because all profits are the income of workers and real income reflects what incomes are worth in consumption.

Real wages are defined by $\omega_{i,t} = \frac{w_{i,t}}{P}$, where P describes the perfect price index such that it recognizes the changes in costs and quality across all sectors. Since nominal wages are unchanging in the model, real wages are increasing at the rate that the economy-wide price index declines. So far, price indices have been defined for each sector only. The perfect price index describes the overall cost of living. It is the P that buys one unit of Q, $P = \frac{1}{N}\prod_{i=1}^{N}\left(P_i^{\frac{1}{N}}\right)$, where P_i is each sector's price index given by Equation 4a.

The rate of technology improvement in a Cournot sector is $g_{i,A} = \frac{A_{i,j,t}-A_{i,j,t-1}}{A_{i,j,t-1}} = \frac{\varepsilon(\sigma_i-1)(n_i+\varepsilon-1)}{\eta(\sigma_i(\varepsilon+1)+n_i-1)} - 1$. The rate of technology improvement in a sector characterized by Bertrand is: $g_{i,A} = \frac{\varepsilon(\sigma_i-1)(n_i+\varepsilon-1)}{\eta(n_i+\varepsilon)} - 1$. The sector i price index is falling at a rate of $(g_{i,A})^{\sigma-1}\frac{1}{1-\sigma}$. Therefore the growth rate of consumption/output is given by the rate that the perfect price index declines:

$$g_Q = \frac{1}{N(1-\sigma)}\prod_{i=1}^{N}(g_{i,A})^{\sigma-1} \tag{19}$$

Since the other portion of economic output, innovation, is constant in this model, the growth rate of total output for each sector is the growth rate of consumption of each sector multiplied by the proportion of the specialized workforce employed in manufacturing. As each sector represents an equal $1/N$ share of the value of total consumption, GDP growth is the average growth rate of all sectors.

GDP growth depends upon the specific make-up of the economy. In particular, the form of competition in each market, the size of each specialized labor supply and the level of contestability. These factors determine the share of employment in innovation or manufacturing and the rate of technology improvement in each sector.

4 Numerical examples

The numerical examples in this section vary the supply of specialized labor as a proxy for contestability in each sector to understand the relationship between contestability and innovation. This conveniently isolates the impact of limited contestability by adjusting only the labor supply on a sector basis which determines the number of participating firms under the discrete entry barrier and also allows for a fair comparison between Bertrand and Cournot competition. Furthermore, the use of a growth model without scale effects or inverse scale assumptions (Bond-Smith et al., 2018) means any differences between sectors' investment in innovation is exclusively a result of contestability under the discrete entry barrier. The analysis compares innovation outcomes in relation to mode of competition and extent of contestability, drawing implications for different economies, industries, regions and countries, and does not represent calibrated estimates of specific economic outcomes.[7]

4.1 Parameter choices

Model parameters are based on typical calibrations using CES preferences. Krugman (1991) uses an elasticity of $\sigma = 4$ and Baldwin and Forslid (2000) uses $\sigma = 5$. Since the model here is disaggregated into many sectors, varieties in the same sector can be expected to be closer substitutes than these calibrations imply. This justifies a higher elasticity of substitution, however, an extremely high σ means consumers are more responsive to changes in quality such that the elasticity overwhelms other factors to determine the quality target. On the otherhand, reducing σ reduces the difference between Cournot and Bertrand, although the conclusions remain unchanged. Therefore, a fixed elasticity of $\sigma = 10$ is chosen for all simulations. While the parameter is higher than Krugman (1991) or Baldwin and Forslid (2000), it is not so high as to overwhelm calculations and is sufficiently high to demonstrate clear differences between models of oligopoly. A sensitivity analysis of σ does not affect the the conclusions drawn.[8]

The rate of time preference follows Baldwin and Forslid (2000) with $\alpha = \frac{1}{2}$, which implies an annual discount rate of approximately 7% when periods represent

[7] Calibrating the model to a specific real world example is beyond the scope of this article as it requires a detailed empirical foundation for calibration.

[8] The magnitude of difference between Cournot and Bertand increases as σ increases, but the relationships remain the same so long as products are assumed substitutes ($\sigma > 1$).

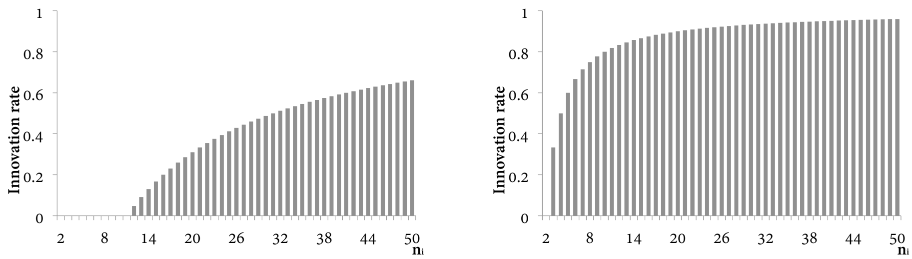
10 years. It results in a technology improvement rate of 1 in the continuous model which enables a simple comparison. Alternative calibrations for α only change the rate of innovation in the steady state. η is calibrated such that the expected free entry rate of technology improvement equals the rate of time preference, i.e. $\eta = \frac{(\sigma-1)}{2} = 12$. γ only has the effect of adjusting the scale in L and its calibration does not affect results, but the same γ is used for comparing Bertrand and Cournot sectors. Initial values for A_i are set to one such that growth in technology can be easily compared to initial technology levels.

4.2 Results

4.2.1 Number of firms and innovation

The following figures describe the relationship between the number of participating firms in a sector and innovation. As the number of firms in a sector rises, innovation increases with each additional firm due to competitive pressure, as firms respond less to their own effect on the price index, because they become a smaller overall share of the sector. These results are supported by a positive empirical relationship between market size, the number of firms and the intenstity of competition (Asplund and Sandin, 1999). The relationship between innovation and contestability can also be considered in the figures as supply of the factor of production (labor) required for entry is directly related to the number of firms. As the availability of the factor of production rises, contestability increases. As a result, innovation increases stepwise, with each step due to the addition of another discrete participating firm. This result is comparable to Navas and Licandro (2011), who find that increasing competition from trade liberalization increases innovation under Cournot oligopoly, but the applies generally to contestability and to both Cournot and Bertrand oligopoly. Furthermore, the model here can closely examine differences between these modes of competition.

Figure 1 describes the technology improvement rates for contestability levels that enable participation by $n_i = 2$ to 50 firms in sectors under Cournot and Bertrand competition respectively. For comparison, the technology improvement rate expected under the continuous model is one. As the number of firms increases, innovation rates tend towards the expected rate under the fully contestable or continuous model. With the assumption of discrete entry, innovation rates never quite reach the continuous limit such that there are always costs to the market from the discrete barrier to entry. Under Cournot competition, for low to moderate levels of contestability, there is zero growth, as simply paying the minimum fixed cost without any quality improvement is enough to deter entry of the marginal firm. Notably under this calibration, it is not until there are 12 firms in the sector before there is positive growth. Notably this seems to be an absurd calibration as 12 firms in a Cournot sector appears to be unnecessarily large and extremely competitive on price, but this emphasises the protection that Cournot competition provides to participating firms such that entrepreneurs do not need to improve quality in order to continue receiving dividends from monopolistic rents.



(a) Cournot competition

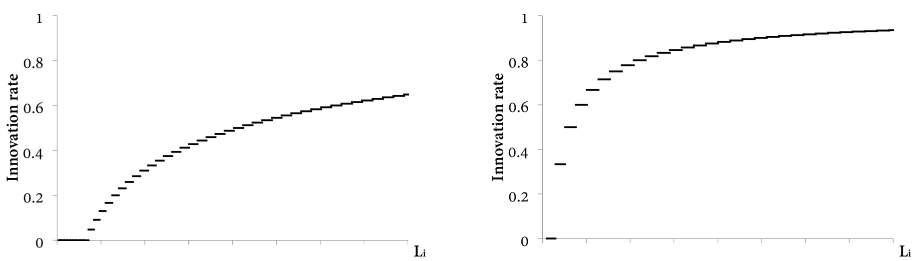
(b) Bertrand competition

Figure 1: Number of discrete firms vs innovation rate

4.2.2 Barriers to entry and innovation

However, this does not tell the full story. The comparison on a firm level basis can be misleading, because firms in Bertrand and Cournot sectors require different levels of the factor of production for entry. While fewer entrepreneurs are required per firm in Cournot competition than in Bertrand competition, the resource constraint barrier allows many more entrants under Cournot than under Bertrand competition. For example, using the same calibration as above, a labor supply that results in 5 firms (and an innovation rate at 0.33) in a Bertrand sector results in 12 firms (and an innovation rate of 0.05) if it were a Cournot sector.

Using the same calibration as above, Figure 2 considers the innovation rates in relation to the labor supply, which operates as a proxy for contestability. Innovation rates rise with increases in the supply of the critical factor of production: labor. Both charts have the same scale in L_i.



(a) Cournot competition

(b) Bertrand competition

Figure 2: Labor supply vs innovation rate

Innovation rates increase stepwise as additional labor allows an additional firm to contest the market at each "step". Each step is discrete and tends towards the growth rate of the continuous model. These steps upwards are initially larger in

Bertrand competition resulting in higher innovation rates for the same labor supply (keeping all other parameters the same). But these higher rates occur with substantially fewer firms in Bertrand competition, meaning the two alternatives are not as substantially different as it appears in Figure 1. Comparing the innovation rates under Bertrand and Cournot competition, the trend towards the continuous innovation rate is much closer when compared on a labor supply (or contestability) basis rather than a number of firms basis.

For policy makers, the mode of competition is particularly important when considering the effect of competition on innovation rates. Firms that compete fiercely on price in sectors under Bertrand competition may be particularly innovative, even with only a few firms, while Cournot sectors could contain many firms but not achieve high rates of innovation. While these Cournot markets may appear competitive with sufficient possibility for entry based on traditional competition analysis and the number of market participants, a lack of competition may be more visible if rates of innovation are examined.

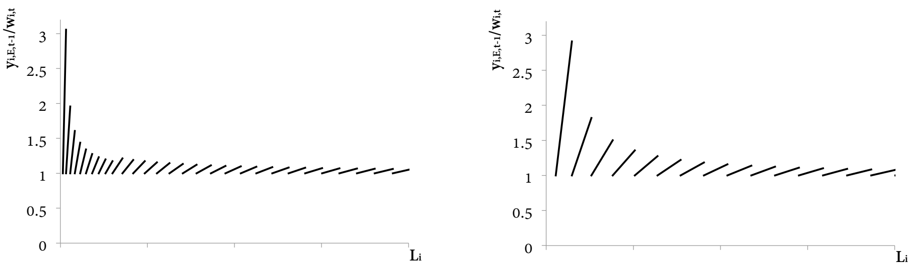
4.2.3 Innovation and wage inequality

Continuing with this calibration, it is possible to also consider how the relationship between innovation and income inequality is affected by varying levels of contestability. The impact of the discrete entry barrier on income inequality varies according to the extent that the marginal firm is prevented from participating by the limited supply of specialized labor. When the labor supply barely allows a marginal firm to participate, entrepreneurs earn exactly the same as workers in manufacturing and the entrepreneurial dividend diminishes to zero. However, if the labor supply prevents entry of a marginal firm to any degree, those workers who are lucky enough to be employed as entrepreneurs earn substantially more than manufacturing workers through their entrepreneurial dividend. The extent of this range of income inequality declines as the labor supply increases.

Figure 3 describes the ratio of entrepreneurs' income including dividends to the wage of manufacturing workers' wages as the labor supply increases under both Cournot and Bertrand competition respectively.[9] While the barrier to entry enables those firms permitted entry to exercise market power over prices and to withhold innovative effort, the extent that this is reflected in the entrepreneurial wage or shared with all workers depends upon the extent that the barrier to entry is binding upon a marginal entrant. In this way there are two processes affecting wage inequality. Firstly, the ability to appropriate monopoly rents and limit innovation declines as contestability increases. But secondly, the extent that the payoff is shared with other employees decreases until a step change occurs with an additional discrete firm. As a result, the relationship between inequality and barriers to entry is ambiguous because it depends upon very specific industry characteristics.

For very low contestability (or labor supply), the resulting level of inequality

[9] The scale in L_i is increased from Figure2.



(a) Cournot competition

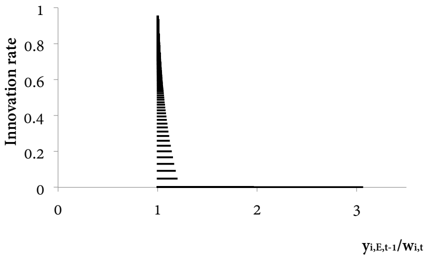
(b) Bertrand competition

Figure 3: Labor supply vs ratio of entrepreneurial income to the market wage

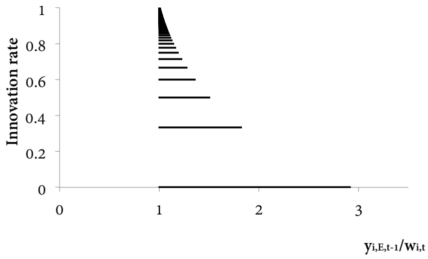
varies substantially, but the range diminishes as contestability increases. Notably, the range of wage inequality increases in a single step in the Cournot sector at the contestability level when growth commences. This is because the labor supply committed to innovation given by Equation 11 is constant when there is no growth at low levels of contestability, but profit increases at the contestability level where innovation results in a quality improvement. Notably the steps are larger under Bertrand competition than under Cournot because Cournot allows many more firms to enter for the same resource constraint.

Using this comparison, it is ambiguous whether the range of inequality is larger under one form of competition or the other. For example, the range of wage inequality for Cournot competition with only two firms is greater than for Bertrand competition with only two firms, but, this direct comparison is also misleading because the potential size of the labor supply over that range is substantially different. With a higher labor supply, the range of inequality diminishes faster for Cournot competition than for Bertrand, as the impact of additional competition in Cournot has a greater impact on entrepreneurial income. As a result, for most levels of contestability, Cournot results in a lower range of income inequality than Bertrand, because many more firms are permitted to enter.

Alternatively, Figure 4 compares innovation rates and wage inequality at various levels of the labor supply under Cournot and Bertrand competition respectively. While the rate of quality improvement in both Cournot and Bertrand eventually converges to one, this does not quite occur even within the 2 to 400 firm range used in these figures. Notably, the upper limits of wage inequality are negatively correlated with the rate of quality improvement because high inequality and low innovation are both a result of barriers to entry or low contestability. As in the above analysis, the relationship between inequality and innovation is ambiguous because it depends upon specific industry characteristics.



(a) Cournot competition



(b) Bertrand competition

Figure 4: Innovation rate vs ratio of entrepreneurial income to the market wage

5 Discussion

The discrete entry barrier results in firms investing in innovation in response to actual competitors and taking account of their own impact on the sector. Participating firms do not respond to potential innovations by entrants because the marginal entrant would not be profitable and is blocked from entry. As all participants can ignore the potential innovations of marginal entrants, investment in innovation is substantially withheld. While price differences for each additional discrete firm are insignificant, though not trivial, differences in the innovation investment behavior of firms when contestability is limited is substantial. This section discusses the model in relation to the existing literature and addressing policy implications.

Notably, the existence of a few sectors with low contestability does not have to condemn an economy or region to lower innovation and growth because each sector only makes up a $1/N$ proportion of an economy. If the economy is dominated by such markets then there would be a significantly greater effect. Furthermore, the exact distribution of sectors is important such that average sector conditions are not the important factor, but to what extent the overall economy is made up of sectors with low contestability. Also real world markets would have a more realistic labor supply where at least some skills of the specialized workforce would be transferable to other sectors. If a particular sector suffers from low contestability, the the sector could be expected to shrink, to the benefit of more innovative sectors of the economy, although such skill transfer mechanism would be weak. Low contestability strongly perverts the incentive for market participants to invest in innovation. Improving contestability and expanding the transferable skills of the labor force is likely to lead to improved innovation performance.

5.1 Contestability, competition, innovation and growth

The most closely related research is Acemoglu and Cao (2015) where the market is contestable for entrants with a drastic innovation. Similarly Klette and Kortum (2004) aim to understand the persistence of incumbent innovation. Acemoglu and

Akcigit (2012) find that an optimal intellectual property rights system should offer greater protection for technology leaders that are far ahead of rivals because this adds to the incentive for innovation to contest and expand the frontier. Innovations allow firms to deter entry by keeping ahead of rivals to escape entry or competition by innovating (Aghion et al., 2001; Acemoglu et al., 2006; Aghion et al., 2009; Griffith et al., 2009; Acemoglu and Akcigit, 2012; Acemoglu and Cao, 2015). Nonetheless, these models all assume *free* entry. But assuming free entry results in the implicit assumption that technological opportunity is available to any potential entrant, provided their investment is adequate. Yet empirical evidence suggests that the important factor determining the relationship between market structure and innovation is the extent of technological opportunity (Scherer, 1967; Levin et al., 1985; Hashmi, 2013) On this basis, the limit on contestability in this article is more consistent with these empirical findings because it considers such a limit on the extent of technological opportunity for potential entrants.

The model developed and derived in this article adds to the existing literature on growth and innovation by examing relationships between innovation and contestability particularly with respect to the mode of competition, number of firms, barriers to entry, inequaltiy and growth. Such an analysis is not possible with standard models of innovation and growth that assume free entry. The results of the model add nuance to understanding the relationship between market structure and innovation or growth because removing contestability by a marginal entrant removes the competition *for* the market effect, leaving only a competition *in* the market effect. In this way the results relate to the upward sloping portion of the inverted U relationship between competiton and innovation found in Aghion et al. (2005). Their research describes a relationship between technology-leading and following firms where competition discourages following firms from innovating, but encourages leading firms who are attempting to "escape competition". Escaping competition in the Aghion et al. (2005) model comes in two forms. Firstly, firms develop innovations to ensure that they are the technology leading firm in the coming period. This is known as the "escape competition" *for* the market effect. The model here also includes the "escape competition" effect but it is only in its second form, that is, competition *in* the market, because of the discrete entry criterion. Discrete entry eliminates the escape competition *for* the market effect because the marginal firm is restricted from entry, but maintains the escape competition *in* the market effect. The downward sloping portion of the inverted U-relationship is therefore not seen because discrete entry prevents the escape competition *for* the market effect from ever dominating the escape competition *in* the market effect.

The model is able to bring the discrete entry assumption and minimum viable scale from the industrial organisation literature (Bain, 1956; Reiss and Spiller, 1989; Berry, 1992) into aggregate models of growth in a similar way to how general equilibrium models of trade have already benefited from such insights (Eaton et al., 2013; Spencer and Brander, 1983; Brander and Spencer, 1985). The findings are consistent with research on the impact of deregulation which highlights how removal or simplification of regulatory barriers (Djankov et al., 2002; Bertrand and

Kramarz, 2002; Alesina et al., 2005; Djankov, 2008) and trade liberalization (Tybout et al., 1991; Pavcnik, 2002; Melitz, 2003; Trefler, 2004; Melitz and Ottaviano, 2008; Verhoogen, 2008; Navas and Licandro, 2011) has stimulated investment.

Similarly the model here is consistent with the approach taken in Desmet and Parente (2010) whereby larger markets increase competition and facilitate innovation. A larger market supports increased product variety and increases the observed price elasticity of demand. Examining the results closer reveals a similar conclusion because both the number of participating firms and their size are also larger in the more contestable markets in order to amortise R&D costs over greater levels of production. Our results could also be extended to include economies of scope as in the model in Desmet and Parente (2010) where research costs are also amortized over more varieties. However, free entry in Desmet and Parente (2010) means the effect on innovation is from competition only while the model here adds the additional dimension of contestability.

The model also highlights how the mode of competition is a particularly important market characteristic for innovation. Low contestability in a Cournot sector results in a substantially lower innovation. When comparing Cournot and Bertrand innovation on a production factor basis (labor), Cournot sectors have considerably lower innovation rates, even though Cournot competition allows significantly more firms to participate. These findings are supported by Navas and Licandro (2011) who examine trade liberalization and innovation, concluding that international rivalry is positive for innovation and that Bertrand oligopoly provides a stronger incentive for innovation than Cournot. Interestingly, the comparison between competition modes here is more nuanced by considering contestability rather than competition. As a result, Cournot oligopoly allows many more firms to enter than industries characterized by Bertrand oligopoly for the same resource constraint because Cournot entrepreneurs withhold a larger share of profit. Yet the incentive provided by additional entry does not compensate entirely for the effect of strategic responses in markets governed by Cournot oligopoly.

5.2 The additional market failure for innovation

Inter-temporal knowledge spillovers lead to a market failure for innovation in endogenous growth models where investors in R&D do not invest at the socially optimal level, because they cannot keep all of the future benefits from the new knowledge that is generated by their innovations (Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990). Those benefits accrue to future innovators when the value of past knowledge is appropriated by new innovators. By assuming free entry, the only dynamic market failure identified by the continuous model is connected with intertemporal knowledge spillovers.

This is also present in the model here, but there is an additional market failure such that firms further under-invest in innovation because they do not respond to potential competitors since the marginal participant is blocked from entry. The market failure is evident in both lower innovation rates and the labor market distortions that cause wage inequality. Examining Figures 1 and 2, the size of the additional market failure for innovation in each sector is the difference between the level of quality improvement achieved and the quality improvement in the model with continuous free entry at one. While the continuous free entry model assumes that a large number of entrants has made this negligible, in the disaggregated model, this additional market failure can be substantial.

The size of the market failure is determined by the extent of contestability under the discrete barrier to entry and by the mode of competition. In Bertrand competition, even if a firm is one of only a few competitors, its effect on the perceived elasticity may be limited. Adding firms to the sector results in the perceived elasticity quickly tending towards the CES elasticity that is observed when there is continuous free entry. However, in Cournot competition, the perceived elasticity tends towards the growth outcome in the continuous free entry model at a much slower rate as contestability increases. The effect of the discrete entry barrier is mitigated because many more firms can enter the market under Cournot competition than under Bertrand competition using the same factors of production. Nonetheless, pressure from additional entry is not enough to entirely mitigate the Cournot effect on innovation. Therefore, innovation is always lower under Cournot competition than Bertrand competition in comparable sectors. Therefore, the size of the innovation market failure is also greater under Cournot competition.

5.3 Regional or isolated economies

In particular, the model's findings appear most relevant for relatively small or isolated regional markets where entry may be more difficult or there is only the capacity for a few firms to enter and such entry criteria could affect a larger share of the economy. Characteristics such as population settlement patterns and geographic isolation (Battersby, 2007) may help to explain rates of innovation in these regions (Wilkie and McDonald, 2008). It is possible that the type of barriers described are also more common in relatively small or isolated regions, where import competition can be relatively more expensive due to economies of scale in transport, there is only capacity for a few firms at a viable scale or where cultural and nationalistic barriers may make entry by foreign firms difficult. Therefore, the model provides new and interesting implications for innovation and economic growth policy that are particularly relevant to small, isolated or peripheral regions and countries. The size of these markets may not be attractive to a multi-national or new entrant such that many subsectors in these economies can remain isolated from the competitive forces that occur in larger agglomerated economies. As a result, there is a positive empirical relationship between market size, the number of firms and the intenstity of competition (Asplund and Sandin, 1999). In this way, the model provides further evidence for localized and industry-specific innovation and growth policy, particularly in small, isolated, non-tradables markets or markets with significant economies of scale where sufficient discrete entry is more likely to be an issue. There is no one-size-fits-all approach to stimulating innovation and growth.

Economic policy must be tailored to an industry, country and region's specific characteristics.

Policy-makers examining competition and innovation policy should consider both the extent of contestability and the mode of competition. Each geographic market and industry will have unique characteristics that determine the effect on innovation and appropriate policies. In response to whether entrants are sufficient or have a minimum viable scale, policy makers should focus on removing or reducing entry requirements, perhaps by offering assistance to new entrants with greater assistance for entry in Cournot sectors. This would also increase the threat of entry and motivate incumbents to innovate.

Trade increases the scale of subsectors by integrating regional economies into larger trading blocs, increasing contestability by increasing the size of the market. International competition therefore increases the threat of entry and stimulates innovation. This implies that isolated or regional economies can reduce their isolation and increase innovation by maintaining open economies and participating in multi-national trade agreements.

5.4 Policy implications

Growth models, either directly or implicitly, suggest conclusions about innovation and macroeconomic growth policy. Over an entire economy, it may be possible for a few or many (or somewhere in between) sectors to have low levels of contestability but the overall effect on economy-wide growth depends upon the portion of the economy made up of sectors with low contestability. The existence of some low contestability sectors does not always have a substantial effect on the economy-wide growth rate. Each sector may makes up a small portion of expenditure and unless the economy is dominated by sectors with barriers to entry, the effect on growth is not necessarily substantial. By including additional microfoundations such as contestability, the model incorporates a market failure for entry and innovation at a sector level and makes a strong link between effective microeconomic policy and economic growth. The intuitive implications from growth models typically suggests macroeconomic policies such as R&D support. In addition, this model shows why it is important to not only consider growth policy as a macroeconomic problem that requires economy-wide solutions, but growth policy should incorporate microeconomic reform in individual sectors.

The intuitive policy response is to examine the factors affecting the innovation cost function and policies that adjust these factors can stimulate innovation. For example, contestability can be expanded by increasing the particular factor of production which is constraining discrete entry. For example, if taxi licenses were the particular factor of production that blocks a marginal entrant, expanding the number of taxi licenses could be expected to increase contestability in the market and encourage innovation by market participants. Alternatively, adjusting how taxi licenses are allocated and minimizing the administrative component could allow such licenses to be contested more easily and avoid strategic cartels from further limiting contestability. Similarly, increasing investment in R&D infrastructure or otherwise to reduce individual firm requirements for that particular factor of production that constrains entry would also expand contestability and encourage innovation.

However, these intuitive policy solutions fail to address the particular problem of a discrete entry barrier. The structural entry barrier still exists but its effects are diminished by these "weak form" policies. There are other "strong form" policies which break the discrete nature of entry, allowing smaller firms to enter, foreign entry or other ways to expand contestability by removing the discrete entry barrier altogether. For example, the removal of license requirements, trade barriers or other minimum scale requirements enables new and smaller firms to enter and contest the market with incumbents.

The discrete entry assumption has implications for merger analysis by offering nuance to policy makers' understanding of the relationship between market structure and innovation. In particular Davis (2003) points out how the relationship between market structure and innovation performance is "far more problematic than in the case of price competition". So modelling developments such as discrete entry that offer this nuance allows anti-trust enforcement to also consider the nature of market structure, rather than simply the number of incumbent firms.

This type of approach is not new to some industries where market failures are clear. Regulation has enabled industries such as telecommunications or electricity to overcome the economies of scale required for entry. These industries are well known as natural monopolies because duplication of the network is expensive, preventing entry of a rival network. In terms of the model developed in this article, access to the network can be thought of as the critical factor of production required for entry into the competitive market for telecommunications services. Telecommunications regulation typically fosters a "ladder of investment" approach (Cave, 2004, 2006) that enables contestability by regulating access to unbundled services such that entrants can be established with a minimal level of investment and gradually expand with higher order levels of investment over time. Lowering the minimum viable scale or the threshold to achieve sufficiency, enables a contest to emerge. Similarly, the regulated separation of retail and wholesale electricity from network services enables contestability for some elements of the electricity supply chain without the scale required for firms to investment in a network. While industry regulation typically targets the natural monopoly nature of these industries, with respect to the model in this article, regulation also removes the discrete entry barrier, thereby encouraging innovation as well as fostering competition on price. It is possible that other markets also suffer from the insufficient contestability. Policy approaches that enable new forms of market entry are therefore likely to improve contestability and stimulate innovation in a variety of industries.

Alternatively, markets can find ways to develop new tools on their own, for alternate forms of entry that overcome the discrete entry barrier. For example, eBay and Amazon have enabled even very small firms to establish a national or even global online sales presence in a number of niche retail markets. Amazon and

Google both enable small businesses to utilise cloud computing without the investment required for their own data centres. Similarly, other firms such as AirBnB or Uber have bypassed entry barriers for taxi services altogether by developing products in a way that does not require the particular factor of production that causes the constraint on entry.

In this way, markets with a discrete entry barrier are also likely to be an attractive target for these types of innovations that overcome entry barriers. Even so, these barriers constrain entry and encourage investment in certain innovations that would be unnecessary if the original market design enabled contestability. These innovations are possible because they extract some of the monopoly rents that would otherwise accrue to incumbents. While markets can sometimes overcome entry barriers, this type of innovation investment would be unnecessary if markets were already sufficiently contestable. Similarly, all of these examples may introduce network effects that risk a reduction in future contestability. Regulators and policy makers should consider whether the barrier or factor of production itself is unnecessary and whether investment in these types of innovations should be required or not in order to overcome any contestability issues. At the same time, regulators should monitor whether new innovations introduce contestability issues and respond accordingly.

6 Conclusion

Using a simple discrete entry barrier (i.e. only an integer number of firms is permitted) as a tool to model contestability, this article develops a framework for examining the impact of market structure on innovation, entrepreneurship and growth. This offers a generalized understanding of how limiting contestability reduces innovation, can increase inequality and how sectors characterized by Cournot face a greater constraint on innovation than sectors characterized by Bertrand, with important implications for regional, competition and innovation policy. When markets are not contestable then entrepreneurs who gain entry can extract monopoly rents with limited competition on price or innovation. However, the ability for entrepreneurs to appropriate rents depends upon the extent that contestability is binding upon a marginal firm. The model here provides a unique insight into this ambiguous and nuanced relationship between regional market structure and innovation.

The model provides a particularly useful framework for examining the relationship between market structure, innovation, entrepreneurship and growth by combining the partial equilibrium characteristics of individual sectors such as barriers to entry, imperfect markets and Cournot or Bertrand oligopoly with the broader general equilibrium features of endogenous growth theory. This encourages a revision of our understanding of endogenous growth to also consider microeconomic and industry characteristics. The model shows that when contestability is limited, innovation and entrepreneurship is constrained because the entry barrier leaves only the "escape competition *in* the market effect". As a result, firms are able to invest in innovation only in response to actual market participants rather than all potential entrants contesting the market. This re-examination of the relationship between contestability and innovation suggests that effective innovation growth policy should address the causes of market structure that have flow-on effects for entrepreneurial investment and income inequality which are masked by traditional competition analysis of market concentration or prices. Furthermore, simple extensions of the model could be used in future research to examine how other microeconomic characteristics impact innovation, entrepreneurship, innovation and growth.

Models that examine the relationship between competition and innovation will benefit from understanding the causes of market structure. The ability to contest the market cannot always be assumed, especially in a regional context. Contestability characteristics are unique to each individual sector and region such that policies to stimulate growth should target innovation or firm entry in these individual sectors and regions. Innovation and growth policy does not necessarily require an economy-wide generic policy approach, but is perhaps a localised, industry- and economy-specific policy problem. Fields such as industrial organization, labor economics, economic geography, and regional and urban economics have much to contribute to the study of local or industry factors affecting entry, contestability, entrepreneurship, innovation and subsequently economic growth. In a similar way, the growth model here adds to the growing body of evidence that innovation policy should focus on localized and industry-specific factors.

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Appendix A Discrete entry

The discrete entry assumption is straightforward and common to the industrial organisation literature (Bain, 1956). Similarly, minimum viable scale or analysis of whether potential entry is sufficient is now standard in approval of mergers and acquisitions. Reiss and Spiller (1989) and Berry (1992) both examine price competition in airline markets where the incumbent firms price strategically to deter discrete entrants. These microfoundations may be missing from existing endogenous growth models. Models with discrete entry are also not completely new to general equilibrium.[10] For example, Eaton et al. (2013) use discrete entry to explain international trade patterns. Similarly, discrete entry is a characteristic of a series of trade models that investigate export subsidies to a domestic producer to deter entry of a foreign business (Spencer and Brander, 1983; Brander and Spencer, 1985). The approach is well-grounded in the industrial organization and trade literatures so extending it to a growth model of innovation based on the same microfoundations is therefore an intuitive and relevant step that yields interesting and elegant insights.

Critical factors of production

This approach could be applied with any critical factor of production required for entry. Specific critical factors of production for entry in various markets could include scarce natural resources, network infrastructure, sector specific skills, entrepreneurial skills, access to finance or collateral, or any other scarce factor of production that is required for market participation. This article provides for a comprehensive model that encompases these specific factors by using a single factor (labor) in order to provide a general examaination of the relationship between limited contestability and innovation. Modelling specific factors of production could be included in any extension of the model developed in this article, but the same implications for contestability would arise.

An example of such a factor of production that could be included in the model would be entrepreneurial skill. The formation and expansion of small and medium sized businesses in developing economies face specific barriers such as a scarcity of entrepreneurial skill and a lack of access to credit (Lloyd-Ellis and Bernhardt, 2000). A model of entrepreneurial skill would imply a heterogeneous labor force and a sorting mechanism by skill, rather than luck, between employment in manufacturing or entrepreneurship. The scarcity of entrepreneurial skill could be examined by varying the distribution of skills within this heterogeneous labor force. Alternatively, adding a heterogeneous distribution of wealth and requiring collateral for entry would allow the model to examine the impact of access to credit on contestability, innovation and growth. Given the analysis of wage inequality in

[10] This model was developed following a discussion with Sam Kortum who used a trade model with an integer number of firms to explain the zeros found in trade data while maintaining other desirable model characteristics (Eaton et al., 2013).

Section 4.2.3, a similar approach that models the distribution of managerial skill could be calibrated help to explain the the ratio between the earnings of CEOs and employees.

Other entry-limiting factors of production or specific barriers to entry could also be added. For example, it may be a regulatory requirement for firms to hold a license to offer products in a particular market, such as taxi services. In this case, the holder of a license would receive a premium as part of a firm's fixed cost, pinning down the number of firms to the number of available licenses. The price of other factors of production would then fall until each factor market clears. Technologies such as Uber, essentially removed the constraint on this key factor of production to increase contestability and consequently innovation. In more complex examples, a cartel of businesses could hold market power such that only a limited number of entrants, including the cartel can participate. This type of strategic response, is an example of how businesses can limit contestability for the critical factor of production required for entry. Natural monopolies such as telecommunications or electricity distribution are often identified as a barrier to entry. In these markets the network can be thought of as the crticial factor of production required for entry and since duplication of the network is expensive, the scarcity of alternative networks prevents the entry of a rival. Regulatory policy that offers entrants access to the underlying network such as local loop unbundling or open access networks increases contestability, thereby stimulating innovation as in the model here. For the purpose of developing conclusions with respect to contestability in general, such specific barriers are considered outside the scope of the model and left for future research, but the conclusions from the model would apply in the same manner.

Lastly, the model could be considered as an example of how contestability might affect isolated or distant markets, where the local market size only allows a limited number of entrants. In these markets the scarce factor of production is a result of local economic geography that impacts on regional productivity (Battersby, 2007). For example, relatively low population density, distances between major cities, distance to major trading partners and a lack of economies of scale might help to explain a lack of competitive pressure in Australia (Wilkie and McDonald, 2008; Minifie et al., 2017). Such implications are also supported by a positive empirical relationship between market size, the number of firms and the intenstity of competition (Asplund and Sandin, 1999).

Contestability versus 'free' entry

Innovation studies suggest that the critical factor of production for innovation is technological opportunity (Scherer, 1967; Levin et al., 1985; Hashmi, 2013), but theoretical models typically assume free and continuous entry. Modern modelling techniques with free entry overlook the degree to which markets are contestable. To allow for barriers to entry, endogenous growth models sometimes use a distance to frontier approach (Acemoglu et al., 2006; Aghion et al., 2009) such that entrants require additional investment in order to match frontier technology. These theoretical models assume that investors expect to earn an additional return for this otherwise arbitrary investment required for entry, but otherwise still assume contestability and free entry. Similarly, the threat from frontier entrants influences incumbent responses to escape competition by innovating themselves (Aghion et al., 2001, 2009; Acemoglu et al., 2006; Griffith et al., 2009), but these models also impose an arbitrary cost to access technology and otherwise assume free entry. While research on the impact of deregulation has highlighted how removal or simplification of regulatory barriers (Djankov et al., 2002; Bertrand and Kramarz, 2002; Alesina et al., 2005; Djankov, 2008) and trade liberalization (Tybout et al., 1991; Pavcnik, 2002; Melitz, 2003; Trefler, 2004; Melitz and Ottaviano, 2008; Verhoogen, 2008; Navas and Licandro, 2011) has stimulated investment, the general relationship between contestability, innovation and growth has not been adequately explored.

The model's findings are supported by Navas and Licandro (2011) who examine trade liberalization and innovation, concluding that international rivalry is positive for innovation and that Bertrand oligopoly provides a stronger incentive for innovation than Cournot. Interestingly, the comparison between competition modes here is more nuanced by considering contestability rather than competition. As a result, Cournot oligopoly allows many more firms to enter than industries characterized by Bertrand oligopoly for the same resource constraint because Cournot entrepreneurs withhold a larger share of profit. Yet the incentive provided by additional entry does not compensate entirely for the effect of Cournot.

The most closely related research is Acemoglu and Cao (2015) where the market is contestable for entrants with a drastic innovation. Similarly Klette and Kortum (2004) aim to understand the persistence of incumbent innovation. Acemoglu and Akcigit (2012) find that an optimal intellectual property rights system should offer greater protection for technology leaders that are far ahead of rivals because this adds to the incentive for innovation to contest and expand the frontier. Nonetheless, these models all assume *free* entry. *This article develops an understanding of how innovation and growth affected when markets are not contestable.*

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